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Time-dependent Gross-Pitaevskii equation for composite bosons as the strong-coupling limit of the fermionic broken-symmetry random-phase approximation

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The linear response to a space- and time-dependent external disturbance of a system of dilute condensed composite bosons at zero temperature, as obtained from the linearized version of the time-dependent Gross-Pitaevskii equation, is shown to result also from the strong-coupling limit of the time-dependent BCS (or broken-symmetry random-phase) approximation for the constituent fermions subject to the same external disturbance. In this way, it is possible to connect excited-state properties of the bosonic and fermionic systems by placing the Gross-Pitaevskii equation in perspective with the corresponding fermionic approximations.

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Ultracold atomic Fermi gases are currently being intensively studied experimentally, as their low-temperature properties can shed light on fundamental questions regarding degenerate Fermi systems. Producing a paired superfluid is a particular challenge, which is being achieved at the present time [1]. Since the attractive interaction responsible for superfluidity in a Fermi gas can be tuned to reach the strong-coupling condition, it also appears possible to study experimentally the evolution from superfluid fermions to condensed composite bosons. (These form as bound-fermion pairs in the strong-coupling limit of the fermionic attraction.) This would permit for the first time a controlled *crossover* between two basic quantum systems sharing the same spontaneous broken-symmetry (superfluid) behavior.

Dilute condensed bosons have already been studied extensively both experimentally [2] and theoretically [3]. The diluteness condition is matched in almost all current experiments. Correspondingly, the Gross-Pitaevskii (GP) equations [4,5] have proven sufficient to describe the observed phenomena. This is true both for ground-state properties (via the time-independent GP equation) *and* for the dynamical response of the condensate to an external disturbance (via the time-dependent GP equation in its linearized form [6]).

With the aim of connecting properties of superfluid fermions and condensed composite bosons, the time-independent GP equation for the condensate wave function was obtained in a previous paper [7] as the strong-coupling limit of the Bogoliubov–de Gennes equations [8] for superfluid fermions at zero temperature.

In this paper, a corresponding connection is established between the linearized form of the time-dependent GP equation (from which the dynamics of the bosonic condensate can be obtained) and the strong-coupling limit of the linear response for superfluid fermions with a BCS ground state, subject to the same external disturbance. We will explicitly show that the fermionic density and current correlation functions map, in the strong-coupling limit of the fermionic attraction, onto the linear-response results for condensed composite bosons. The fermionic correlation functions are obtained from the time-dependent BCS [or broken-symmetry random-phase approximation (RPA)] approximation, while the linear-response results for the composite bosons are obtained from the time-dependent GP equation. Thus a connection between ground- and excited-state properties of the condensate both in the weak- and strong-coupling limits is established. On physical grounds the connection between the time-independent and time-dependent versions of the GP equation for condensed composite bosons and the BCS and time-dependent BCS approximations for superfluid fermions, rests on the zero-point motion in a quantum theory implying an oscillatory spectrum. The imprint of the quasiparticle spectrum should be found in the ground-state wave function [4].

The response functions of a system of condensed bosons described by the wave function $\Phi(\mathbf{r},t)$ can be obtained from the time-dependent GP equation [4,5]:

$$(1/2m_B)\,[i\boldsymbol{\nabla}+(q_B/c)\,\mathbf{A}_{\text{ext}}(\mathbf{r},t)]^2\Phi(\mathbf{r},t)+V_{\text{ext}}(\mathbf{r},t)\Phi(\mathbf{r},t)+U_0|\Phi(\mathbf{r},t)|^2\Phi(\mathbf{r},t)=i\{[\partial\Phi(\mathbf{r},t)]/\partial t\} \quad (1)$$

in the presence of space- and time-dependent vector $\mathbf{A}_{\text{ext}}$ and scalar V_{ext} external potentials. Here, c is the velocity of light, m_B and q_B are the mass and charge of the bosons, and $U_0=4\pi a_B/m_B$ is the short-range repulsive potential expressed in terms of the bosonic scattering length a_B ($\hbar=1$ throughout). Note that we have generalized the time-dependent GP equation to include a vector potential, as a mere *formal* device to obtain, in addition to the density-density correlation function already considered for neutral bosons [6], the current-density and current-current correlation functions. In this way we can compare these functions with the corresponding quantities for fermions obtained within the time-dependent BCS approximation in the strong-coupling limit. Accordingly, no Coulomb repulsion between bosons is included.

The linear-response solution of Eq. (1) for $\mathbf{A}_{\text{ext}}=0$ is obtained [6] by taking $V_{\text{ext}}(\mathbf{r},t)=V_{\mathbf{q}}e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)}+V_{\mathbf{q}}^{*}e^{-i(\mathbf{q}\cdot\mathbf{r}-\omega t)}$ with wave vector $\mathbf{q}$ and frequency ω, and by setting to linear order

$$\Phi(\mathbf{r},t)=\sqrt{n_B}[1+\overline{u}_{\omega}(\mathbf{r})e^{-i\omega t}-\overline{v}_{\omega}(\mathbf{r})^{*}e^{i\omega t}]e^{-i\mu_B t}. \quad (2)$$

n_B is the ground-state bosonic density, $\mu_B=U_0 n_B$ the bosonic chemical potential, and the "small" components $\overline{u}_{\omega}(\mathbf{r})$ and $\overline{v}_{\omega}(\mathbf{r})$ are correspondingly taken of the form $\overline{u}_{\omega}(\mathbf{r})=\overline{u}_{\mathbf{q},\omega}e^{i\mathbf{q}\cdot\mathbf{r}}$ and $\overline{v}_{\omega}(\mathbf{r})=\overline{v}_{\mathbf{q},\omega}e^{i\mathbf{q}\cdot\mathbf{r}}$. The time-dependent

GP equation (1) then reduces to a linear system for the unknowns $\overline{u}_{\mathbf{q},\omega}$ and $\overline{v}_{\mathbf{q},\omega}$. The solution is

$$\overline{u}_{\mathbf{q},\omega}=\frac{\omega+E_B^0(\mathbf{q})}{\omega^2-E_B(\mathbf{q})^2}V_{\mathbf{q}}, \quad \overline{v}_{\mathbf{q},\omega}=\frac{\omega-E_B^0(\mathbf{q})}{\omega^2-E_B(\mathbf{q})^2}V_{\mathbf{q}}. \tag{3}$$

$E_B^0(\mathbf{q})=\mathbf{q}^2/(2m_B)$ is the free-boson energy and $E_B(\mathbf{q})=\sqrt{E_B^0(\mathbf{q})^2+2\mu_B E_B^0(\mathbf{q})}$ the Bogoliubov dispersion relation. The induced particle density and current are then obtained by expanding the expressions $n(\mathbf{r},t)=|\Phi(\mathbf{r},t)|^2$ and $\mathbf{j}(\mathbf{r},t)=\mathrm{Im}[\Phi(\mathbf{r},t)^*\boldsymbol{\nabla}\Phi(\mathbf{r},t)/m_B]$ to linear order in V_{ext}. One obtains

$$\delta n(\mathbf{r},t)=\frac{n_B}{m_B}\,\frac{\mathbf{q}^2}{\omega^2-E_B(\mathbf{q})^2}V_{\mathrm{ext}}(\mathbf{r},t), \tag{4}$$

$$\mathbf{j}(\mathbf{r},t)=\frac{n_B}{m_B}\,\frac{\mathbf{q}\omega}{\omega^2-E_B(\mathbf{q})^2}V_{\mathrm{ext}}(\mathbf{r},t). \tag{5}$$

These satisfy the continuity equation $\partial\delta n(\mathbf{r},t)/\partial t+\boldsymbol{\nabla}\cdot\mathbf{j}(\mathbf{r},t)=0$. By linear-response theory, the density-density and current-density correlation functions can be identified as

$$\chi_{nn}(\mathbf{q},\omega)=(n_B/m_B)\{\mathbf{q}^2/[\omega^2-E_B(\mathbf{q})^2]\}, \tag{6}$$

$$\chi_{\mathbf{j}n}(\mathbf{q},\omega)=(n_B/m_B)\{\mathbf{q}\omega/[\omega^2-E_B(\mathbf{q})^2]\}, \tag{7}$$

with the typical Bogoliubov form.

Similarly, the linear-response solution of Eq. (1) when $V_{\mathrm{ext}}=0$ is obtained by taking $\mathbf{A}_{\mathrm{ext}}(\mathbf{r},t)=\mathbf{A}_{\mathbf{q}}e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)}+\mathbf{A}_{\mathbf{q}}^*e^{-i(\mathbf{q}\cdot\mathbf{r}-\omega t)}$ and using the same form (2) for the wave function. Working in a gauge where $\boldsymbol{\nabla}\cdot\mathbf{A}_{\mathrm{ext}}(\mathbf{r},t)$ is nonzero (such that $\mathbf{q}\cdot\mathbf{A}_{\mathbf{q}}\neq 0$) and retracing the above arguments, one obtains

$$\overline{u}_{\mathbf{q},\omega}=\frac{E_B^0(\mathbf{q})+2\mu_B+\omega}{E_B(\mathbf{q})^2-\omega^2}\,\frac{q_B}{2m_Bc}\mathbf{q}\cdot\mathbf{A}_{\mathbf{q}}, \tag{8}$$

$$\overline{v}_{\mathbf{q},\omega}=\sqrt{n_B}\,\frac{E_B^0(\mathbf{q})+2\mu_B-\omega}{E_B(\mathbf{q})^2-\omega^2}\,\frac{q_B}{2m_Bc}\mathbf{q}\cdot\mathbf{A}_{\mathbf{q}} \tag{9}$$

in place of Eq. (3). The ensuing induced density and current now take the form:

$$\delta n(\mathbf{r},t)=\frac{n_B}{m_B}\,\frac{\omega\mathbf{q}}{\omega^2-E_B(\mathbf{q})^2}\cdot\left(-\frac{q_B}{c}\mathbf{A}_{\mathrm{ext}}(\mathbf{r},t)\right), \tag{10}$$

$$\mathbf{j}(\mathbf{r},t)=\frac{2n_B}{(2m_B)^2}\mathbf{q}\frac{E_B^0(\mathbf{q})+2\mu_B}{\omega^2-E_B(\mathbf{q})^2}\mathbf{q}\cdot\left(-\frac{q_B}{c}\mathbf{A}_{\mathrm{ext}}(\mathbf{r},t)\right). \tag{11}$$

These replace Eqs. (4) and (5), with the continuity equation being satisfied once the diamagnetic contribution $-(q_Bn_B)/(m_Bc)\mathbf{A}_{\mathrm{ext}}(\mathbf{r},t)$ is added to the induced current (11). The density-current and current-current correlation functions also acquire the typical Bogoliubov form:

$$\chi_{n\mathbf{j}}(\mathbf{q},\omega)=(n_B/m_B)\{\omega\mathbf{q}/[\omega^2-E_B(\mathbf{q})^2]\}, \tag{12}$$

$$\chi_{\mathbf{jj}}(\mathbf{q},\omega)=(n_B/m_B^2)\,\mathbf{q}\{[\mu_B+E_B^0(\mathbf{q})/2]/[\omega^2-E_B(\mathbf{q})^2]\}\,\mathbf{q}. \tag{13}$$

The correlation functions (6), (7) and (12), (13) describe the linear response of the bosonic condensate to an external disturbance in terms of Bogoliubov quasiparticle excitations.

We now show that the same form of the correlation functions is obtained from the fermionic time-dependent BCS (or broken-symmetry RPA) approximation in the strong-coupling limit of the fermionic attraction. We describe the fermionic attraction by a short-range (contact) potential regularized in terms of the two-body scattering length a_F [9,10]. By increasing the strength of the attraction, a bound state appears in the two-body problem and a_F becomes positive. For strong attraction, the fermionic chemical potential μ approaches $-\varepsilon_0/2$, where $\varepsilon_0=(ma_F^2)^{-1}$ is the binding energy of the composite bosons which form in this limit. The composite bosons have mass m_B twice the fermionic mass m and *positive* scattering length $a_B=2a_F$. The positive scattering length ensures the stability of the bosonic system and results from the Pauli repulsion between the constituent fermions. (It was first pointed out in Ref. [10] that the relation $a_B=2a_F$ is actually an approximate one, being the result of the BCS mean-field treatment of the fermionic attraction. More refined treatments yield a reduction of a_B by about a factor of 3 [10,11]. This reduction, however, is not expected to be relevant to the main results of this paper.)

We first express the fermionic correlation functions in the broken-symmetry phase in terms of the two-particle correlation function

$$L(1,2;1',2')=\langle T_\tau[\psi(1)\psi(2)\psi^\dagger(2')\psi^\dagger(1')]\rangle - \mathcal{G}(1,1')\mathcal{G}(2,2'). \tag{14}$$

Here, $\langle\cdots\rangle$ is a thermal average, T_τ the time-order operator for imaginary time τ, $1=(\mathbf{r}_1,\tau_1,\ell_1)$ with the index $\ell=(\mathrm{I,II})$ identifying the components of the field operator in the Nambu representation $[\psi_{\mathrm{I}}(\mathbf{r})=\psi_\uparrow(\mathbf{r})$ and $\psi_{\mathrm{II}}(\mathbf{r})=\psi_\downarrow^\dagger(\mathbf{r})]$, and $\mathcal{G}(1,2)=-\langle T_\tau[\psi(1)\psi^\dagger(2)]\rangle$ is the single-particle Green's function. The two-particle correlation function (14) can be expressed in terms of the many-particle T matrix [12]

$$-L(1,2;1',2')=\mathcal{G}(1,2')\mathcal{G}(2,1')+\int d3456\,\mathcal{G}(1,3)\mathcal{G}(6,1')\times T(3,5;6,4)\mathcal{G}(4,2')\mathcal{G}(2,5). \tag{15}$$

An explicit form for T results from the Bethe-Salpeter equation once a choice for its kernel is made.

Within the (off-diagonal) BCS approximation we are considering, the Bethe-Salpeter equation for T can be explicitly solved in the frequency and wave-vector representation. For a short-range (contact) potential, the only nonvanishing elements of the many-particle T matrix correspond to the

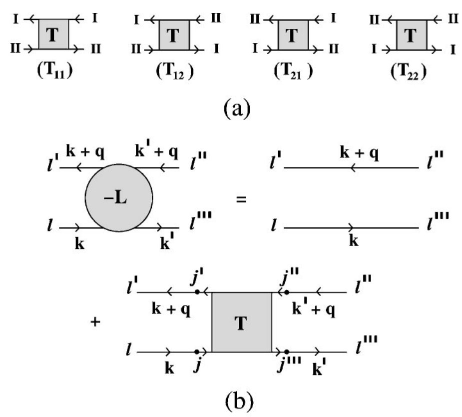

FIG. 1. (a) Nambu indices associated with the four nonvanishing elements of the many-particle T matrix for a fermionic contact potential; (b) Relation between the two-particle correlation function L and the many-particle T matrix.

Nambu indices reported in Fig. 1(a). In the strong-coupling limit the binding energy ε_0 becomes the largest energy scale, thus yielding $\beta\mu \to -\infty$ and $\Delta \ll |\mu|$ (β being the inverse temperature and Δ the BCS gap function). In this limit, one thus obtains the following expressions for the nonvanishing elements of the many-particle T matrix

$$T_{11}(\mathbf{q},\Omega_\nu) = T_{22}(\mathbf{q},-\Omega_\nu) \simeq \frac{8\pi}{m^2 a_F}\,\frac{i\Omega_\nu + \mathbf{q}^2/(2m_B) + \mu_B}{(i\Omega_\nu)^2 - E_B(\mathbf{q})^2}, \tag{16}$$

$$T_{12}(\mathbf{q},\Omega_\nu) = T_{21}(\mathbf{q},\Omega_\nu) \simeq -\frac{8\pi}{m^2 a_F}\,\frac{\mu_B}{(i\Omega_\nu)^2 - E_B(\mathbf{q})^2}. \tag{17}$$

$\Omega_\nu = 2\nu\pi/\beta$ (ν integer) is a bosonic Matsubara frequency [13], $\mu_B = 2\mu + \varepsilon_0$ is the same bosonic chemical potential introduced after Eq. (2), and $E_B(\mathbf{q})$ coincides formally with the Bogoliubov dispersion of Eq. (3). The fermionic single-particle Green's functions are correspondingly given by the BCS expressions

$$\mathcal{G}_{11}(\mathbf{k},\omega_n) = -\mathcal{G}_{22}(\mathbf{k},-\omega_n) = -\frac{\xi(\mathbf{k}) + i\omega_n}{E(\mathbf{k})^2 + \omega_n^2}, \tag{18}$$

$$\mathcal{G}_{21}(\mathbf{k},\omega_n) = \mathcal{G}_{12}(\mathbf{k},\omega_n) = \frac{\Delta}{E(\mathbf{k})^2 + \omega_n^2}. \tag{19}$$

$\omega_n = (2n+1)\pi/\beta$ (n integer) is a fermionic Matsubara frequency [13], $\xi(\mathbf{k}) = \mathbf{k}^2/(2m) - \mu$, and $E(\mathbf{k}) = \sqrt{\xi(\mathbf{k})^2 + \Delta^2}$ for an (s-wave) isotropic gap function Δ.

The fermionic correlation functions for the number density and current can be quite generally obtained from their definitions as thermal averages of the time-order operator acting on the number density and current operators. This is done by expressing the number density and current operators in terms of Nambu field operators and introducing the two-particle correlation function (14) accordingly. One ends up with the following expressions:

$$\chi_{nn}(q) = -\sum_{\ell,\ell',\ell'',\ell'''} \tau^3_{\ell,\ell'}\tau^3_{\ell'',\ell'''} \int \frac{d\mathbf{k}\,d\mathbf{k}'}{(2\pi)^6}\,\frac{1}{\beta^2} \times \sum_{n,n'} L^{\ell',\ell''}_{\ell,\ell'''}(k,k';q), \tag{20}$$

$$\chi_{n\mathbf{j}}(q) = -\frac{1}{2m}\sum_{\ell,\ell',\ell'',\ell'''} \tau^3_{\ell,\ell'}\tau^0_{\ell'',\ell'''} \int \frac{d\mathbf{k}\,d\mathbf{k}'}{(2\pi)^6} \times (2\mathbf{k}'+\mathbf{q})\,\frac{1}{\beta^2}\sum_{n,n'} L^{\ell',\ell''}_{\ell,\ell'''}(k,k';q), \tag{21}$$

$$\chi_{\mathbf{jj}}(q) = -\frac{1}{(2m)^2}\sum_{\ell,\ell',\ell'',\ell'''} \tau^0_{\ell,\ell'}\tau^0_{\ell'',\ell'''} \int \frac{d\mathbf{k}\,d\mathbf{k}'}{(2\pi)^6} \times (2\mathbf{k}+\mathbf{q})(2\mathbf{k}'+\mathbf{q})\,\frac{1}{\beta^2}\sum_{n,n'} L^{\ell',\ell''}_{\ell,\ell'''}(k,k';q). \tag{22}$$

The notation is $q = (\mathbf{q},\Omega_\nu)$, $k = (\mathbf{k},\omega_n)$, $k' = (\mathbf{k}',\omega_{n'})$, where τ^0 and τ^3 are Pauli matrices, and the labels for L are specified in Fig. 1(b). The first term on the right-hand side of the diagrammatic representation of Fig. 1(b) corresponds to the standard BCS contribution to the response functions [13]. This term vanishes in the strong-coupling limit for temperatures much below the dissociation temperature of the composite bosons. The second term, on the other hand, corresponds to the broken-symmetry BCS-RPA approximation and is required to maintain full gauge invariance [14]. In the present context this term has a nontrivial strong-coupling limit and yields the response of the composite bosons condensate. From this term one obtains for the correlation functions (20)–(22)

$$\chi_{nn}(q) = \sum_{\ell,\ell',\ell'',\ell'''} D_{\ell,\ell'}(q)\,T^{\ell',\ell''}_{\ell,\ell'''}(q)\,D_{\ell''',\ell''}(q), \tag{23}$$

$$\chi_{n\mathbf{j}}(q) = \sum_{\ell,\ell',\ell'',\ell'''} D_{\ell,\ell'}(q)\,T^{\ell',\ell''}_{\ell,\ell'''}(q)\,\mathbf{V}_{\ell''',\ell''}(q), \tag{24}$$

$$\chi_{\mathbf{jj}}(q) = \sum_{\ell,\ell',\ell'',\ell'''} \mathbf{V}_{\ell,\ell'}(q)\,T^{\ell',\ell''}_{\ell,\ell'''}(q)\,\mathbf{V}_{\ell''',\ell''}(q), \tag{25}$$

with

$$D_{\ell,\ell'}(q) \equiv \sum_{\ell'',\ell'''} \tau^3_{\ell'',\ell'''} \int \frac{d\mathbf{k}}{(2\pi)^3}\,\frac{1}{\beta}\sum_n \mathcal{G}_{\ell,\ell''}(k) \times \mathcal{G}_{\ell''',\ell'}(q+k), \tag{26}$$

$$\mathbf{V}_{\ell,\ell'}(q) \equiv \frac{1}{2m} \sum_{\ell'',\ell'''} \tau^0_{\ell'',\ell'''} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{\beta} \sum_n (2\mathbf{k}+\mathbf{q}) \times \mathcal{G}_{\ell,\ell''}(k) \mathcal{G}_{\ell''',\ell'}(q+k). \tag{27}$$

Recalling that, for a fermionic contact potential, only the four elements (16) and (17) of the many-particle T matrix are nonvanishing, and exploiting the symmetry properties of the quantities (26) and (27), one arrives at

$$\chi_{nn}(q) = (D(q), D(-q)) \begin{pmatrix} T_{11}(q) & T_{12}(q) \\ T_{21}(q) & T_{22}(q) \end{pmatrix} \begin{pmatrix} D(q) \\ D(-q) \end{pmatrix},$$

$$\chi_{n\mathbf{j}}(q) = (D(q), D(-q)) \begin{pmatrix} T_{11}(q) & T_{12}(q) \\ T_{21}(q) & T_{22}(q) \end{pmatrix} \begin{pmatrix} \mathbf{V}(q) \\ \mathbf{V}(-q) \end{pmatrix},$$

$$\chi_{\mathbf{jj}}(q) = (\mathbf{V}(q), \mathbf{V}(-q)) \begin{pmatrix} T_{11}(q) & T_{12}(q) \\ T_{21}(q) & T_{22}(q) \end{pmatrix} \begin{pmatrix} \mathbf{V}(q) \\ \mathbf{V}(-q) \end{pmatrix},$$

where $D(q) \simeq -4m\Delta(ma_F)/(16\pi)$ and $\mathbf{V}(q) \simeq -\mathbf{q}\Delta(ma_F)/(16\pi)$. Using the approximate expressions (16) and (17), one obtains finally for the fermionic correlation functions in the strong-coupling limit

$$\chi_{nn}(q) \simeq D^2(q=0)[T_{11}(q) + T_{11}(-q) + 2T_{12}(q)] = 4(n_B/m_B)\{\mathbf{q}^2/[(i\Omega_\nu)^2 - E_B(\mathbf{q})^2]\}, \tag{28}$$

$$\chi_{n\mathbf{j}}(q) \simeq D(q=0)\mathbf{V}(q)[T_{11}(q) - T_{11}(-q)] = 4(n_B/m_B)\{i\Omega_\nu \mathbf{q}/[(i\Omega_\nu)^2 - E_B(\mathbf{q})^2]\}, \tag{29}$$

$$\chi_{\mathbf{jj}}(q) \simeq \mathbf{V}(q)\mathbf{V}(q)[T_{11}(q) + T_{11}(-q) - 2T_{12}(q)] = 4\frac{n_B}{m_B^2}\mathbf{q}\frac{\mu_B + \mathbf{q}^2/(4m_B)}{(i\Omega_\nu)^2 - E_B(\mathbf{q})^2}\mathbf{q}. \tag{30}$$

The analytic continuation $i\Omega_\nu \to \omega + i\delta$ ($\delta = 0^+$) to the real frequency ω is now required to obtain the retarded correlation functions. In this way the bosonic expressions (6),(12), and (13) are recovered from their fermionic counterparts in the strong-coupling limit. There is an additional factor of four, reflecting the fact that the bosonic number and current densities are half the fermionic values.

The present mapping has been explicitly established for fermions (and composite bosons) in the absence of a static external potential. On physical grounds we expect the connection we have established between the time-dependent GP equation and the fermionic broken-symmetry BCS-RPA approximation to remain valid also in the presence of a static confining potential.

A similar connection between the time-dependent GP equation in strong coupling and the time-dependent Ginzburg-Landau equation in weak coupling has been established in Ref. [9] at the level of the functional integral in the absence of a static external potential.

The above forms of the correlation functions obtained either from the time-dependent GP equation or from the strong-coupling limit of the fermionic broken-symmetry BCS-RPA approximation, contain *only* the contribution of the condensate and thus hold at or near zero temperature. Since the current response of the condensate can be only longitudinal, the transverse current correlation function cannot be obtained in this way since it requires inclusion of excitations out of the condensate. These excitations are important for determining the behavior of the system at finite temperature and close to the superfluid transition temperature. Corrections to the time-dependent GP equation, or equivalently additional fermionic contributions to the current correlation function in the broken-symmetry phase over and above the BCS-RPA approximation, are thus required to recover, for instance, the Bogoliubov approximation to the current correlation function for bosons [15]. This contains a transverse in addition to a longitudinal part.

In conclusion, we have shown that in the strong-coupling limit of the mutual fermionic attraction the response functions obtained at zero temperature from the linearized form of the time-dependent GP equation for a system of composite bosons are equivalent to those obtained within the broken-symmetry BCS-RPA approximation for the original fermionic system.

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