

Magnetic Field Effect on the Pseudogap Temperature within Precursor Superconductivity

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We determine the magnetic-field dependence of the pseudogap closing temperature T^* within a precursor superconductivity scenario. Detailed calculations with an anisotropic lattice model with d -wave superconductivity account for a recently determined experimental relation in BSCCO between the pseudogap closing field and the pseudogap temperature at zero field, as well as for the weak initial dependence of T^* at low fields. Our results indicate that the available experimental data are fully compatible with a superconducting origin of the pseudogap in cuprate superconductors.

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High-temperature superconductors are characterized by a *pseudogap phase*, whereby a temperature T^* for the closing of the pseudogap is identified in addition to the (lower) superconducting critical temperature T_c . The origin of this pseudogap phase remains controversial and has mainly been ascribed to two alternative scenarios, namely, to the occurrence of a “competing order parameter” (due, e.g., to an underlying antiferromagnetic or charge order) or to the presence of “precursor superconductivity” (due to strong pairing attraction above T_c). The issue is thus to decide whether pseudogap and superconductivity (better, precursor pairing) represent related or different phenomena.

In this context, magnetic effects are good candidates for distinguishing between the two scenarios. It turns out that T^* and T_c behave differently as functions of hole concentration and magnetic-field H , in the sense that, in the underdoped region, T^* is weakly dependent on H while T_c shows more pronounced dependence [1]. A similar behavior is observed at optimal doping [2], while in the overdoped region the pseudogap behavior is strongly field dependent [3]. The above distinction between the two temperature (or energy) scales has sometimes been quoted as evidence that the two scales have different physical origin [4,5].

Recently, by applying magnetic fields up to 60 T, a systematic determination of the pseudogap closing field H_{pg} (above which the pseudogap phase is destroyed at any temperature) has been made using data for several doping values [6], showing strikingly different dependencies from the characteristic fields of the superconducting state. In addition, it has been found [6] that H_{pg} and T^* at zero field are related (within experimental uncertainty) through the simple Zeeman-like expression $g\mu_B H_{pg} = k_B T^*$, where $g = 2$ is the electronic g -factor, μ_B the Bohr magneton, and k_B the Boltzmann constant. This expression suggests that the magnetic-field couples to the pseudogap by the Zeeman energy of the spin degrees of freedom, thus entailing a predominant role of the spins over the orbital frustration effects in the formation of the pseudogap.

Within the framework of the pairing scenario [7,8], a calculation of the effect of a magnetic field on T^* and T_c

has already been presented by including *orbital* magnetic effects only but no Zeeman coupling in a continuum model [9], with the conclusion that orbital effects should contribute in an important way to the field dependence of T^* .

In this Letter, we calculate the magnetic-field dependence of T^* within a precursor superconductivity scenario, by taking explicit *account of the Zeeman splitting for the spin degrees of freedom*, and provide specific comparison with the data of Ref. [6]. To this end, we initially consider the Zeeman splitting only, within an anisotropic lattice model with d -wave superconductivity. At a second stage, we include also orbital effects in a continuum model, to assess the relative importance of Zeeman and orbital couplings. At the doping levels relevant to experiments, we reproduce quantitatively the expression $g\mu_B H_{pg} = k_B T^*$ proposed by the authors of Ref. [6]. Specifically, we find that: (i) Inclusion of the sole Zeeman splitting for an anisotropic lattice model with d -wave superconductivity recovers the above expression within about a factor of 2 (which we attribute to the difference between the temperature T^* as identified within precursor superconductivity and the energy scale associated with pair breaking); (ii) The contribution of the orbital effect (estimated for a continuum model in the intermediate-coupling regime relevant to experiments) does not significantly affect the results obtained for H_{pg} with the sole inclusion of the Zeeman effect, in accordance with the naive expectation of a small orbital pair-breaking effect for a short-coherence length superconductor; (iii) In the small H region, $T^*(H)$ is weakly dependent on H and somewhat sensitive to the inclusion of the orbital effect. Our results imply that the magnetic-field effects in the pseudogap phase, as determined by the experiments reported in Ref. [6] as well as in Refs. [1,2,10] are fully compatible with precursor superconductivity as the origin of T^* .

The relative importance of the Zeeman and orbital effects on H_{pg} could be assessed by applying a magnetic field, alternatively, *parallel* and *orthogonal* to the layered structure of the high-temperature superconducting materials, thus suppressing the orbital contribution when the magnetic field is parallel to the layers. Although no

experimental indication in this sense yet exists, the relation between H_{pg} and T^* found in Ref. [6] constitutes *per se* an empirical suggestion that the Zeeman effect dominates over the orbital frustration, at least as far as H_{pg} is concerned.

For the above reasons, we consider initially the Zeeman effect only, which can be evaluated for arbitrary magnetic field and for which lattice effects (even with anisotropy) and the d -wave character of precursor pairing can be readily included. To this end, we consider a lattice model with nearest-neighbor (t_{\parallel}) and next-nearest neighbor (t'_{\parallel}) in-plane hoppings, nearest-neighbor (t_{\perp}) out-of-plane hopping, and an in-plane attraction (with strength $V \leq 0$) between nearest-neighbor sites. This model is believed to capture the essential physics of cuprate superconductors in the nearly optimally doped region.

In the absence of magnetic field, a *pair breaking temperature* is obtained by supplementing the standard BCS gap equation with the particle-density equation to determine the chemical potential [11], since its value is no longer pinned at the Fermi level as soon as the attraction is sufficiently strong. When this happens, the above temperature is no longer related to the superconducting critical temperature, but rather signals the formation of (quasi) bound pairs. In this context, the pair-breaking temperature is then associated with the crossover temperature T^* identified experimentally. In the following, we extend this identification to the presence of a magnetic field. The coupled equations to be solved are the following:

$$\frac{1}{|V|} = \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{\gamma(\mathbf{k})^2}{4\xi(\mathbf{k})} \sum_{\sigma} \tanh\left[\frac{\xi_{\sigma}(\mathbf{k})}{2k_B T^*}\right], \quad (1)$$

$$n = \int \frac{d\mathbf{k}}{(2\pi)^3} \sum_{\sigma} \frac{1}{\exp[\xi_{\sigma}(\mathbf{k})/k_B T^*] + 1}. \quad (2)$$

In these expressions, n is the site density, $\xi_{\sigma}(\mathbf{k}) = -2t_{\parallel}(\cos k_x + \cos k_y) - 2t_{\perp} \cos k_z + 4t'_{\parallel} \cos k_x \cos k_y - \mu + \mu_B H \sigma$ (where μ is the chemical potential and $\sigma = \pm 1$), $2\xi(\mathbf{k}) = \xi_+(\mathbf{k}) + \xi_-(\mathbf{k})$, $\gamma(\mathbf{k}) = (\cos k_x - \cos k_y)$ for a d -wave gap, and each component of the wave vector \mathbf{k} is integrated from $-\pi$ to $+\pi$. To make contact with experimental data, we interpret $n = 1 - p$ where p is the doping value [6]. Values appropriate to BSCCO are $t_{\parallel} = 0.25$ eV, $t'_{\parallel} = 0.45t_{\parallel}$, and $t_{\perp} = 0.2t_{\parallel}$ [12]. The value of $|V|$ (corresponding to a given sample) is obtained by solving the above equations for $H = 0$ and setting $T^*(H = 0)$ at the experimental value in zero field. These equations are then solved numerically for the unknowns (T^* , μ) with given values of (n , $|V|$, H). In this way, to any given sample [corresponding to a pair of values (n , $|V|$)], we associate a curve $T^*(H)$, from which the pseudogap closing field H_{pg} of interest is extracted by extrapolation to $T^*(H_{\text{pg}}) = 0$ (extrapolation is required to avoid numerical instabilities when $T^*(H)$ approaches zero).

A typical curve $T^*(H)$ obtained by the above procedure is shown in Fig. 1, for the values of n and $|V|$ corresponding to the sample with $p = 0.21$ analyzed in Ref. [6]. Note the *rather weak initial dependence* of $T^*(H)$ for low H . Typically, $T^*(H)$ decreases only by about 1% for values of H up to 25 T. This result is consistent with the experimental finding reported by several authors [1,2,10], where a negligibly weak magnetic-field dependence of T^* is noted for various compounds.

Figure 2 shows the results of our calculation corresponding to the three samples reported in the inset of Fig. 4 from Ref. [6]. In that reference, the large (≥ 100 T) values of H_{pg} have been obtained by extrapolation of the data collected up to 60 T. Note that both the experimental and the theoretical points lie on straight lines passing through the origin. This entails a *linear proportionality* between H_{pg} and $T^*(H = 0)$ [of the form $2\mu_B H_{\text{pg}} = \alpha k_B T^*(H = 0)$ where α is a constant], as pointed out in Ref. [6]. [This linear proportionality has been established in Ref. [6] not only for the three doping values reported in the inset of their Fig. 4, but also for lower doping values (albeit with larger uncertainty), as shown in their Fig. 4. We have also extended our calculation to the value $p = 0.16$, still obtaining the linear behavior.]

The experimental data give $\alpha_{\text{exp}} \simeq 1$, while our calculation yields $\alpha_{\text{theo}} \simeq 2.4$. This difference should not be regarded as being too severe. First of all, recall that T^* is, by its own nature, a nonsharply defined crossover temperature, so that there is an intrinsic uncertainty in its identification. Nonetheless, one may attribute the above discrepancy to additional physical effects (such as orbital frustration) not included in Eqs. (1) and (2). We have, however, verified for the continuum model (see below) that orbital frustration contributes only in a marginal way to the suppression of T^* in the intermediate-coupling regime. In addition, one may observe that, while the experimental

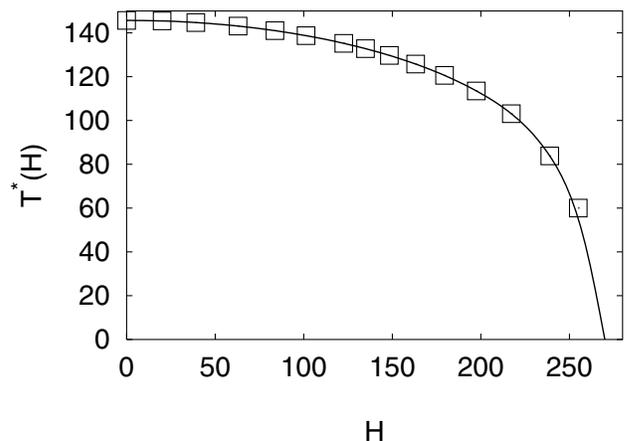


FIG. 1. Dependence of $T^*(H)$ on H obtained from Eqs. (1) and (2) with $n = 0.79$ and $|V| = 0.175$ eV. The continuous curve is a polynomial fit. (Temperature is expressed in Kelvin and magnetic field in Tesla.)

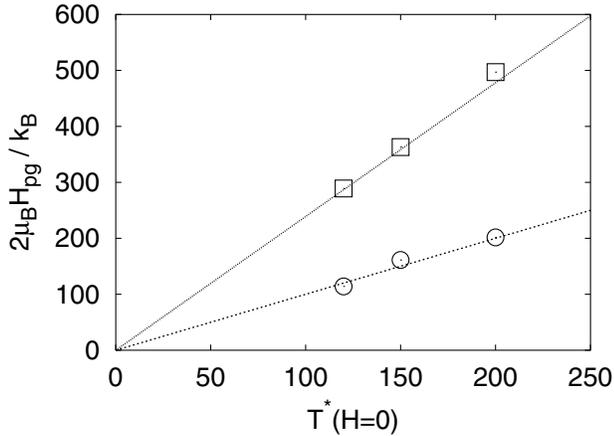


FIG. 2. Theoretical (squares) and experimental (circles) values of $2\mu_B H_{pg}/k_B$ vs $T^*(H=0)$, for the three samples [with $p = (0.20, 0.21, 0.22)$ and $|V|/t_{||} = (0.8, 0.7, 0.65)$ from right to left] reported in the inset of Fig. 4 from Ref. [6]. (Temperatures are expressed in Kelvin.)

value T_{exp}^* (as obtained in Ref. [6]) identifies directly an energy scale (which, by our interpretation, is associated with the binding energy E_b of a pair embedded in the medium [13]), the theoretical value T_{theo}^* can be *smaller* than E_b . This statement is readily verified in the strong-coupling limit of the continuum model considered below, whereby $k_B T_{\text{theo}}^* = (\epsilon_o/2)/\ln(\epsilon_o/\epsilon_F)^{3/2}$ in zero field [14,15], ϵ_F being the Fermi energy and ϵ_o the binding energy of the associated two-body problem (E_b is expected to reduce to ϵ_o in this limit). From this relationship, one gets $T_{\text{theo}}^* < \epsilon_o$ when $\epsilon_o/\epsilon_F > \exp(1/3) \simeq 1.4$. At weaker (intermediate) couplings, on the other hand, numerical calculation yields $T_{\text{theo}}^* \simeq \epsilon_o/2$ when $\epsilon_o/\epsilon_F \simeq 1$. In this case, the experimental relation $g\mu_B H_{pg} = k_B T_{\text{exp}}^*$ is thus equivalent to the theoretical relation $g\mu_B H_{pg} = \alpha_{\text{theo}} k_B T_{\text{theo}}^*$ with $\alpha_{\text{theo}} \simeq 2$ (provided one still identifies E_b with ϵ_o). The difference between the experimental and theoretical identification of T^* may thus account for the difference between α_{exp} and α_{theo} .

In any event, the physical point to be emphasized from our calculation is that the *energy scales* associated with H_{pg} and $T^*(H=0)$ are of the *same order*, as found experimentally. The experimental result thus finds a natural explanation within our precursor superconductivity scenario, whereby $T^*(H=0)$ is associated with the binding energy of the (quasi) preformed pairs and H_{pg} with the Pauli pair breaking due to Zeeman splitting.

To assess the role of orbital frustration, it is sufficient to analyze the simpler continuum model for which both Zeeman and orbital effects can be readily included. In this way, we avoid facing at this stage the peculiar problems of Bloch electrons in a magnetic field [16–18].

In the continuum case, the fermionic attraction can be modeled by a point contact interaction, regularized (in three dimensions) in terms of the scattering length a_F

[14,19]. The dimensionless interaction parameter $(k_F a_F)^{-1}$ [where the Fermi wave vector k_F is related to the density via $n = k_F^3/(3\pi^2)$] then ranges from $-\infty$ in weak coupling to $+\infty$ in strong coupling [the crossover region of interest being bound by $(k_F |a_F|)^{-1} \lesssim 1$]. For this model, the equations identifying T^* within our approach (for an s -wave pseudogap) and corresponding to Eqs. (1) and (2) are the following:

$$\frac{m}{4\pi a_F} + R(T^*, \mu; H) - \omega_c b(T^*, \mu; H) = 0, \quad (3)$$

$$n = \frac{m\omega_c}{2\pi} \sum_{s=0}^{\infty} \int_{-\infty}^{+\infty} \frac{dk_z}{2\pi} \sum_{\sigma} f \left[\left(s + \frac{1}{2} \right) \omega_c + \frac{k_z^2}{2m} - \mu_{\sigma} \right], \quad (4)$$

where m is the fermion mass, $\omega_c = eH/(mc)$ the cyclotron frequency, $\mu_{\sigma} = \mu - \mu_B H \sigma$, $f(x) = [\exp(x/k_B T^*) + 1]^{-1}$ the Fermi function at temperature T^* , and

$$R(T^*, \mu; H) = \int_0^{\infty} d\epsilon N(\epsilon) \left[\sum_{\sigma} \frac{\tanh(\frac{\epsilon - \mu_{\sigma}}{2k_B T^*})}{4(\epsilon - \mu)} - \frac{1}{2\epsilon} \right], \quad (5)$$

$$b(T^*, \mu; H) = \int_0^{\infty} d\epsilon \frac{N(\epsilon) \epsilon}{6(\mu - \epsilon)^2} \left[\frac{F(\epsilon)}{(\mu - \epsilon)} + F'(\epsilon) \right], \quad (6)$$

$N(\epsilon)$ being the independent-particle density of states per spin component and $F(\epsilon) = f(\epsilon - \mu_+) + f(\epsilon - \mu_-) - 1$. These equations contain both the Zeeman splitting (through the spin dependence of μ_{σ}) and the orbital effect [through the presence of $b(T^*, \mu; H)$ in Eq. (3) and of the Landau levels in Eq. (4)]. These equations include, however, the effect of the magnetic field on a different footing. Equation (3) has been obtained by considering the Thouless instability in the presence of a magnetic field. Specifically, the orbital effect was introduced in the calculation of the particle-particle ladder by adopting the eikonal approximation, whereby the phase factor $\exp\{i\varphi\}$ multiplies the independent-particle Green's function in the absence of the field [20] [with $\varphi = e[\mathbf{A}(\mathbf{r}) + \mathbf{A}(\mathbf{r}')] \times (\mathbf{r} - \mathbf{r}')/(2c)$, $\mathbf{A}(\mathbf{r})$ being the vector potential at position \mathbf{r}]. In this way, the kernel of the integral equation defining the particle-particle ladder is readily diagonalized in the basis of the Landau level wave functions. The ensuing divergence of the particle-particle ladder in the lowest Landau level (and for vanishing frequency and wave vector) yields eventually Eqs. (3) and (4). Equation (4), on the other hand, keeps the Landau orbital quantization explicitly, otherwise the eikonal approximation would completely wash out the magnetic orbital effect.

It is relevant to discuss at this point the conditions for the validity of the eikonal approximation in the present context. The domain of validity of the eikonal approximation for the single-particle Green's function depends on the distance between its two spatial coordinates. For Eq. (3),

this domain is delimited by the range l of the two-particle propagator in zero field, yielding $\omega_c \ll p_G/(ml)$ where p_G is largest momentum scale of the independent-particle Green's function. In addition, by expanding up to second order the phase factor $\exp\{i\varphi\}$ of the eikonal approximation in Eq. (3), one arrives at the additional condition $\omega_c \ll (ml^2)^{-1}$. In weak coupling, $p_G \approx k_F$ and $l \approx \xi_o$ where ξ_o is the Pippard coherence length. In this case, one obtains $\omega_c \ll (m\xi_o^2)^{-1}$ for the validity of the eikonal approximation. In strong coupling, on the other hand, $p_G^{-1} \approx a_F \approx l$, leading to $\omega_c \ll \epsilon_o$ for the validity of the eikonal approximation.

The above estimates lead us to conclude that, strictly speaking, the eikonal approximation holds only for the *initial part* of the curve $T^*(H)$ close to zero field. Taking $\omega_c m \xi_o^2 \lesssim 0.1$ in weak and $\omega_c/\epsilon_o \lesssim 0.1$ in strong coupling, one obtains correspondingly $[T^*(H=0) - T^*(H)]/T^*(H=0) \lesssim 1\%$ and $\lesssim 10\%$. The relative range of validity of the eikonal approximation increases, therefore, when passing from weak to strong coupling. In practice, we shall extrapolate the results for $T^*(H)$ obtained from Eqs. (3) and (4) outside the strict range of validity of the eikonal approximation to obtain the pseudogap closing field H_{pg} , since we are interested in obtaining a qualitative estimate of the relative importance of the Zeeman and orbital effects at intermediate coupling. The eikonal approximation, on the other hand, does not affect the equations determining $T^*(H)$ when the Zeeman effect only is included.

Figure 3 shows $T^*(H)$ vs H obtained by Eqs. (3) and (4) for the continuum model with the value $(k_F a_F)^{-1} = 0.5$ in the intermediate-coupling regime, by including both Zeeman *and* orbital effects (triangles) or the Zeeman effect only (squares) (the temperature T^* has been conveniently normalized in terms of the Bose-Einstein temperature T_{BE} for composite bosons of mass $2m$ and density $n/2$). Note that $\alpha_{\text{theo}} \approx 2.6$ with the Zeeman effect only, while $\alpha_{\text{theo}} \approx 2.3$ when both effects are included (to be compared with the value $\alpha_{\text{theo}} \approx 2.4$ from Fig. 2). This implies that the calculation with the Zeeman effect only is rather stable against the inclusion of the orbital effect. Note also from this figure that, in the small H region, $T^*(H)$ is somewhat sensitive to the inclusion of the orbital effect [albeit the dependence of $T^*(H)$ on H is still weak in an absolute sense], in agreement with the experimental result [10] for fields up to 14 T.

In conclusion, we have shown that consideration of the Zeeman splitting within the precursor superconductivity scenario (at intermediate coupling) reproduces the experimental relation $2\mu_B H_{pg} = k_B T^*(H=0)$ between the pseudogap closing field H_{pg} and the pseudogap temperature $T^*(H=0)$ at zero field [6]. Our results are also fully consistent with the occurrence of a weak initial dependence of $T^*(H)$ on H at low fields [1,2,10]. As the sensitivity to a magnetic field is believed to be a crucial test for the

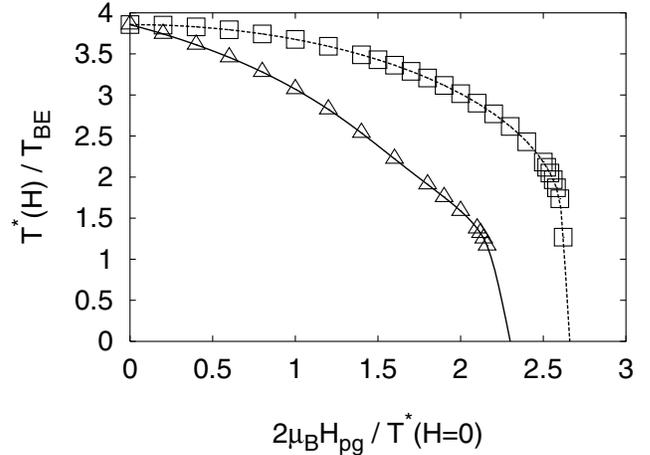


FIG. 3. $T^*(H)$ vs H for the continuum model with coupling value $(k_F a_F)^{-1} = 0.5$, including the Zeeman effect only (squares) and the orbital effect as well (triangles). The continuous curves are polynomial fits.

physical origin of the pseudogap [5], our results show that the available experimental data are fully compatible with a superconducting origin of the pseudogap.

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