Density and Spin Response of a Strongly Interacting Fermi Gas in the Attractive and Quasirepulsive Regime

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Recent experimental advances in ultracold Fermi gases allow for exploring response functions under different dynamical conditions. In particular, the issue of obtaining a ''quasirepulsive'' regime starting from a Fermi gas with an attractive interparticle interaction while avoiding the formation of the two-body bound state is currently debated. Here, we provide a calculation of the density and spin response for a wide range of temperature and coupling both in the attractive and quasirepulsive regime, whereby the system is assumed to evolve nonadiabatically toward the ''upper branch'' of the Fermi gas. A comparison is made with the available experimental data for these two quantities.

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Ultracold Fermi gases represent testing systems for resolving many open issues in condensed and nuclear matter. A key feature of these systems is that the interparticle interaction can be varied with unprecedented flexibility through the use of Fano-Feshbach resonances from the weak- to the strong-coupling limits, which correspond to the presence of correlated and truly bound pairs, in the order. Recent experimental advances have also made it possible to achieve an accurate control of the temperature in such a way that the temperature dependence of several physical quantities can be explored.

In particular, due to the diluteness condition of an ultracold Fermi gas, the temperature interval that can be explored ranges from about 5% of the Fermi temperature T_F up to several times T_F. Such a wide temperature range allows alternative theoretical approaches and the corresponding results to be tested, from the temperature regime $T \gg T_F$, where the first few virial corrections to the free Fermi gas are relevant, down to the temperature region $T \approx T_c$, where the interplay of thermal and quantum fluctuations signals the presence of a superfluid phase that develops at the critical temperature T_c.

In this context, a recent experiment [1] has reported values for the compressibility and spin susceptibility of a unitary Fermi gas over a wide temperature range ($0.2 \lesssim T/T_F \lesssim 10$), setting a benchmark for theoretical calculations that address the (static limits of the) density and spin correlation functions.

Here, we present theoretical results for the compressibility and spin susceptibility, obtained within linear-response theory by a diagrammatic approach built on the t-matrix approximation and its variations. The diagrams selected for the calculation bear on familiar contributions in condensed matter, namely, the density-of-states (DOS), Maki-Thompson (MT), and Aslamazov-Larkin (AL) diagrams [2] that are depicted in Fig. 1.

In the temperature range $T_c \lesssim T \lesssim 5T_F$ relevant to compare with the experimental data, we shall obtain the compressibility as the static limit of the density correlation function, by adding to the DOS diagram of Fig. 1(a) the MT diagram of Fig. 1(b), the AL diagrams of Figs. 1(c) and 1(d), plus the whole series of Fig. 1(e) which is built on these AL diagrams [2]. This is because for the compressibility it is essential to take into account the residual interaction active above T_c among correlated fermion pairs, in order to prevent the compressibility from diverging when

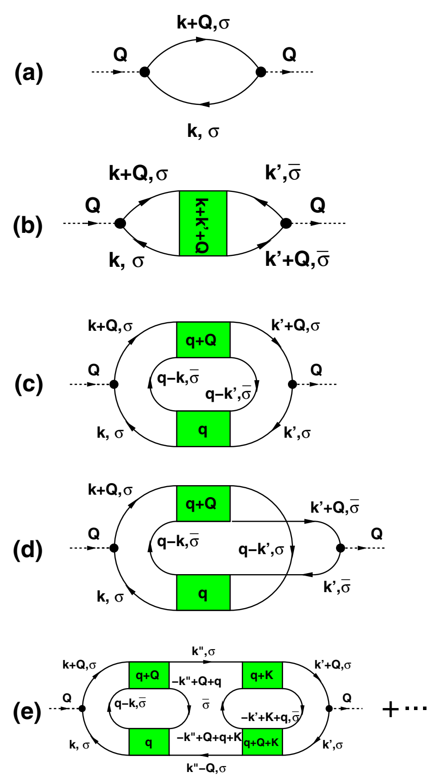

FIG. 1 (color online). (a) DOS, (b) MT, (c) AL, and (d) twisted AL diagrams, plus (e) series of modified AL diagrams, that are used to calculate the compressibility and spin susceptibility. The four vector Q is taken to vanish in the static limit.

approaching T_c similarly to what occurs for pointlike bosons. To this end, it is necessary to improve on the standard t-matrix approach, which has been successfully used in a variety of contexts but would now lead to a diverging compressibility when approaching T_c from above. This is achieved by including the residual interaction via the diagrammatic approach of Ref. [3], which is equivalent to the Popov approach for (composite) bosons in the Bose-Einstein condensation (BEC) limit. The good agreement we will obtain by this approach with the experimental data on the compressibility (see below), over the whole temperature range where they are available, indicates that the residual interaction among correlated fermion pairs above T_c [4] represents a key ingredient for the thermodynamic stability of the system.

For the spin susceptibility, only the DOS diagram of Fig. 1(a) and the MT diagram of Fig. 1(b) are relevant above (as well as below) T_c [2]. On physical grounds, for a Fermi gas with attractive interaction one expects the spin susceptibility to be suppressed when T_c is reached from above and to vanish eventually deep in the superfluid phase when $T \ll T_c$, by the argument that partners in Cooper pairs get locked in a spin singlet. This feature, which can be obtained already at the mean-field level in a purely BCS approach [5], should be even more pronounced by the occurrence of pairing fluctuations above (as well as below) T_c [6], which are associated with the occurrence of a pseudogap. This behavior results also from our calculation for the attractive gas when pairing fluctuations are included, but is not consistent with the experimental data for the spin susceptibility of Ref. [1].

In Ref. [1] the lack of suppression of the spin susceptibility close to T_c was seen as challenging the existence of a pseudogap in the unitary Fermi gas [7]. However, measurements that directly probe the single-particle excitations [8,9] as well as a number of theoretical calculations [10–13] have supported the existence of a pseudogap in the unitary Fermi gas. We shall show that the above apparent contradiction can be resolved by assuming that the measurement of the spin susceptibility of Ref. [1] actually explores a nonequilibrium state associated with a *quasirepulsive* Fermi gas, and therefore cannot be directly compared with equilibrium calculations for an attractive Fermi gas, as remarked already in Ref. [14].

Although the experimental data for the compressibility and spin susceptibility were reported in Ref. [1] only at unitarity, we shall extend our calculations to both sides of the Fano-Feshbach resonance for couplings $(k_F a_F)^{-1} > 0$ on the BEC side and $(k_F a_F)^{-1} < 0$ on the BCS side of the crossover. Here, k_F is the Fermi wave vector related to the density by $n = k_F^3/(3\pi^3)$ and a_F the scattering length. For the attractive gas, this extension to both sides of the crossover is required to compare with the spin susceptibility data reported in Ref. [15] for the trapped gas. For the quasirepulsive gas, one needs to extend the calculation up to $(k_F a_F)^{-1} = 10$ to recover the results of a "dilute" repulsive gas [16] with good accuracy.

Figure 2 shows the temperature dependence of the compressibility at unitarity. The experimental data from Fig. 4(a) of Ref. [1] (circles) are compared with the theoretical results obtained for an attractive Fermi gas from the static limit of the density correlation function χ_n in the normal phase above T_c (in this way, only a couple of experimental data at the lowest T are missed). The calculations neglect (dashed line) or include (full line) the residual interaction among preformed Cooper pairs above T_c and are based, respectively, on the DOS plus MT and AL diagrams and on the DOS plus MT and the *whole series* of AL diagrams of Fig. 1. The results obtained from the high-temperature (virial) expansion of Ref. [17] (dash-dotted line) and those for a noninteracting Fermi gas (dotted line) are also shown for comparison. Note how the inclusion of the residual interaction among preformed Cooper pairs is essential to get meaningful results for the compressibility, which would otherwise diverge when $T \to T_c^+$ within the standard t-matrix approximation [18].

Figure 3 shows the corresponding temperature dependence of the spin susceptibility at unitarity. The experimental data from Fig. 4(a) of Ref. [1] (circles) are compared with the theoretical results obtained for an attractive Fermi gas from the static limit of the spin correlation function χ_s. The results shown by the full line are obtained above T_c by summing the contributions of the DOS [Fig. 1(a)] and MT [Fig. 1(b)] diagrams, and extended to the superfluid phase by adding the fluctuation contributions associated with these diagrams below T_c to the BCS result [5,7] (see also [2]). The results obtained

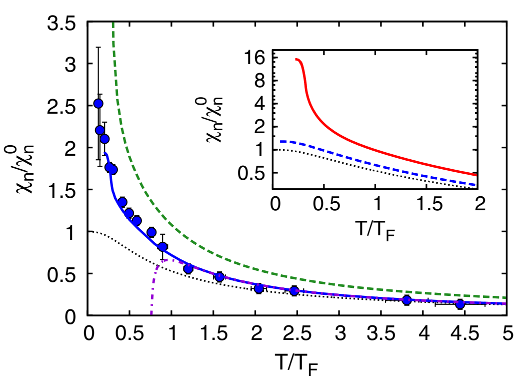

FIG. 2 (color online). Compressibility at unitarity versus T/T_F, normalized by the value for an ideal Fermi gas at $T = 0$. The data from Ref. [1] (circles) are compared with alternative theoretical calculations. Full line: Most complete calculation, including diagrams $a + b + c + d + e$ of Fig. 1; dashed line: calculation including diagrams $a + b + c + d$; dash-dotted line: virial expansion; dotted line: noninteracting Fermi gas. The inset compares the results of our most complete calculation when $(k_F a_F)^{-1} = +1.0$ (full line) and $(k_F a_F)^{-1} = -1.0$ (dashed line) with those of a noninteracting Fermi gas (dotted line).

from the high-temperature (virial) expansion of Ref. [17] (dash-dotted line) and those of a noninteracting Fermi gas (dotted line) are again shown for comparison [19].

Our results reproduce well the virial expansion for an attractive Fermi gas up to $T \approx 5T_F$, and are consistently below those for a noninteracting Fermi gas. This indicates a tendency toward pair formation in the normal state, that at lower temperature leads to a pronounced drop in the spin susceptibility which signals the presence of a "spin gap" well before the onset of the superfluid phase [6]. The spin susceptibility vanishes eventually for $T \to 0$, reflecting Cooper pairing in spin singlets [5,7].

Our results for the attractive Fermi gas, however, show marked deviations from the experimental data of Ref. [1], which lie instead above those for a noninteracting Fermi gas also at high temperature and do not perceive the expected suppression due to the singlet correlation in Cooper pairs at low temperature. This may indicate that the specific dynamical conditions through which the spin susceptibility data were determined in Ref. [1] have *de facto* excluded the formation of the two-body bound state, resulting in the formation of a "quasirepulsive" Fermi gas with an effective *repulsive* interaction [14].

Before passing to describe such a quasirepulsive Fermi gas to connect with the spin susceptibility data of Ref. [1], it may be relevant to compare our theoretical calculations for the spin susceptibility of an attractive Fermi gas with an alternative set of data from Ref. [15], which was taken at thermodynamic equilibrium for a trapped gas across the BCS-BEC crossover. These data are reported in the inset of Fig. 3 (circles) and are in good agreement with our theoretical calculations for the trapped system (diamonds) [the two couplings on the left (right) correspond to a temperature $0.13T_F$ ($0.19T_F$)].

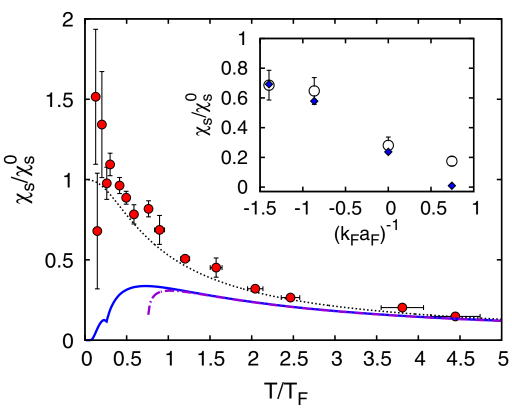

FIG. 3 (color online). Spin susceptibility at unitarity versus T/T_F, normalized by the value for an ideal Fermi gas at $T = 0$. The data from Ref. [1] (circles) are compared with alternative theoretical calculations for an attractive Fermi gas. Full line: Calculation including diagrams $a + b$ of Fig. 1 above T_c and their extension on top of the BCS contribution below T_c; dash-dotted line: virial expansion; dotted line: noninteracting Fermi gas. The inset compares the data of Ref. [15] (circles) for a trapped Fermi gas at equilibrium with our calculations (diamonds).

A route to the description of a repulsive Fermi gas starting from an attractive one was provided recently in Ref. [20], by the exclusion of the bound-state contribution from the density equation on the BEC side of unitarity within a Nozières-Schmitt-Rink approach. It turns out, however, that this approach results in a wide forbidden region of the temperature-coupling phase diagram, in such a way that it cannot be used to obtain the spin susceptibility close to unitarity at low temperature.

Here, we propose an alternative approach to describe the quasirepulsive Fermi gas, which focuses directly on the (particle-particle) ladder propagator Γ_0 entering the response diagrams of Fig. 1 and eliminates the effects of the bound state on the BEC side of unitarity as follows. Let us consider the spectral representation

$$\Gamma_0(\mathbf{q}, i\Omega_\nu) = -\int_{-\infty}^{+\infty} \frac{d\omega}{\pi} \frac{\mathrm{Im}\Gamma_0^R(\mathbf{q}, \omega)}{i\Omega_\nu - \omega} \tag{1}$$

for the ladder propagator of the attractive Fermi gas. Here, $\mathbf{q}$ is the center-of-mass wave vector, $\Omega_\nu = 2\pi\nu T$ (ν integer) a bosonic Matsubara frequency, and $\Gamma_0^R(\mathbf{q}, \omega) = \Gamma_0(\mathbf{q}, i\Omega_\nu \to \omega + i\eta)$ with $\eta = 0^+$. To exclude the contribution of the two-body bound state on the BEC side of the resonance, we need to eliminate from $\mathrm{Im}\Gamma_0^R(\mathbf{q}, \omega)$ the deltalike (polar) contribution at the given $\mathbf{q}$. This is done by starting the ω integration in Eq. (1) at the continuum threshold given by $\omega_c(\mathbf{q}) = \mathbf{q}^2/(4m) - 2\mu$, where m and μ are the fermionic mass and chemical potential, respectively.

An analogous restriction on the frequency integration in the expression of the density is only what was required within the approach of Ref. [20] to get the thermodynamics of the quasirepulsive Fermi gas. Our use of the spectral representation (1), however, requires us to also take into account an additional frequency-independent two-body term, which needs to be subtracted from the many-body ladder propagator in order to reproduce the correct behavior of a weakly repulsive Fermi gas when $(k_F a_F)^{-1} \gg 1$. This frequency-independent term can be inferred from the work of Ref. [21], and eventually yields the following expression for the ladder propagator of a quasirepulsive Fermi gas:

$$\Gamma_0^{\mathrm{rep}}(\mathbf{p}, \mathbf{q}, i\Omega_\nu) = -\int_{\omega_c(\mathbf{q})}^{+\infty} \frac{d\omega}{\pi} \frac{\mathrm{Im}\Gamma_0^R(\mathbf{q}, \omega)}{i\Omega_\nu - \omega} - \frac{8\pi/(ma_F)}{a_F^{-2} + \mathbf{p}^2}, \tag{2}$$

where $2\mathbf{p}$ is the incoming relative wave vector. Note that Eqs. (1) and (2) coincide at unitarity. The occurrence of an energy-independent term is familiar, for instance, in the dispersion relation for the forward scattering amplitude in scattering theory [22]. We are going to use the expression (2) to calculate the thermodynamics and the response

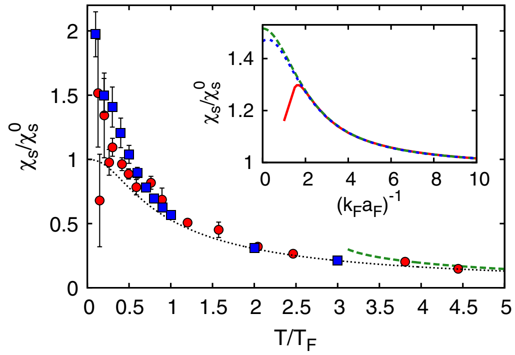

FIG. 4 (color online). Spin susceptibility at unitarity versus T/T_F, normalized by the value for an ideal Fermi gas at $T = 0$. The data from Ref. [1] (circles) are compared with alternative theoretical calculations for a quasirepulsive Fermi gas. Squares: Our extrapolated values for a quasirepulsive Fermi gas; dashed line: virial expansion; dotted line: noninteracting Fermi gas. The inset shows the coupling dependence of the spin susceptibility for a given $T(= 0.2T_F)$ (full line), together with an extrapolation procedure based on two different fitting functions.

functions of this out-of-equilibrium system, provided the density equation admits solutions [18].

In particular, we have found that the spin susceptibility at a given temperature as a function of coupling has the typical shape shown by the full line in the inset of Fig. 4. This shape coincides with that of a truly repulsive dilute Fermi gas [16] when $(k_F a_F)^{-1} \gg 1$, and departs only slightly from it even when the coupling gets reduced down to $(k_F a_F)^{-1} \approx 2$ (this lower coupling turns out to be almost independent of temperature [23]). The spin susceptibility starts then to drop when the coupling is lowered further toward unitarity, approaching a value that corresponds to an attractive Fermi gas at the given temperature (a value that can be reached only for temperatures lying outside the forbidden region in the temperature-coupling diagram).

Our assumption at this point is that, by extrapolating the shape of the spin susceptibility before it drops at $(k_F a_F)^{-1} \approx 2$ in the way shown by the broken lines in the inset of Fig. 4 (corresponding to two different fitting functions), we should end up by reaching an excited configuration as if an avoided level crossing were present, with a dynamics determined by Landau-Zener processes [24]. (Similar ideas were also discussed in Refs. [25,26] while analyzing the competing instabilities towards Stoner ferromagnetism and pairing.) Correspondingly, we assume that the dynamics of the colliding clouds in the experiment of Ref. [1] activates a number of Landau-Zener processes, such that the data there reported for the spin susceptibility at unitarity as a function of temperature can be directly compared with our extrapolated values obtained by the above procedure.

Figure 4 compares our extrapolated values for the quasirepulsive Fermi gas at unitarity (squares) with the data from Fig. 4(a) of Ref. [1] (circles) over an extended temperature range. The error bars attached to our extrapolated values derive from the statistical uncertainties produced by the different fitting functions (like in the inset). On physical grounds, these error bars may be thought of as associated with the underlying presence of a *large* number of Landau-Zener processes mentioned in Ref. [26]. The results obtained by the high-temperature expansion of Ref. [27] where the bound-state contribution has been subtracted off (dashed line) and those of the noninteracting gas (dotted line) are also reported in Fig. 4. The good comparison between the experimental data and our theoretical results supports our treatment of the quasirepulsive Fermi gas as well as the assumptions about the underlying dynamical processes that result in the spin susceptibility data of Ref. [1]. It is further relevant to mention that by our approach to a quasirepulsive Fermi gas there is no evidence for a Stoner instability toward a ferromagnetic phase, in accordance with a recent experimental finding [28].

In conclusion, we have reported theoretical calculations for the compressibility and spin susceptibility over a wide temperature and coupling range and compared them with recent experimental data for Fermi gases. For the compressibility, the residual interaction among fermion pairs turned out to be an essential ingredient for comparing favorably with the experimental data for an attractive Fermi gas. The spin susceptibility was calculated for both an attractive and a suitably defined quasirepulsive Fermi gas, and has been favorably compared with different sets of experimental data in the two cases. For the quasirepulsive gas this comparison has relied on assuming a dynamics determined by Landau-Zener processes.

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