Infrared Behavior of Interacting Bosons at Zero Temperature

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(Received 22 March 1996)

We exploit the symmetries associated with the stability of the superfluid phase to solve the long-standing problem of interacting bosons in the presence of a condensate at zero temperature. Implementation of these symmetries poses strong conditions on the renormalizations that heal the singularities of perturbation theory. The renormalized theory gives the following: For $d > 3$ the Bogoliubov quasiparticles as an exact result; for $1 < d \leq 3$ a nontrivial solution with the exact exponent for the singular longitudinal correlation function, with phonons again as low-lying excitations. [S0031-9007(97)02495-2]

PACS numbers: 03.75.Fi, 11.10.Hi, 67.40.–w

The problem of understanding the low-lying excitations from the ground state of an interacting Bose system has been one of the major problems of condensed matter theory in the Fifties and Sixties. The first solution to this problem was given by Bogoliubov as a generalized Hartree-Fock approximation [1]. Numerous papers were then devoted to analyze the corrections to this approximate solution [2–12]. Apart from approximations showing a gap in the excitation spectrum, all attempts to improve the Bogoliubov approximation encountered the difficulty of a singular perturbation theory (PT) plagued by infrared (IR) divergences, due to the presence of the Bose-Einstein condensate and the Goldstone mode [2,3].

A systematic study of these IR divergences has been long delayed because they appeared only at intermediate steps of the calculations while physical quantities turned out to be finite [2]. That problems could arise in PT was originally recognized by Gavoret and Nozières [3]; indeed later [5] it was found that the exact anomalous self-energy (at zero external momentum) has to vanish [13], in contrast with the finite value obtained by the Bogoliubov approximation. This result questions the validity of PT and requires a properly renormalized theory.

To take care of the IR divergences at arbitrary spatial dimension d greater than 1, we exploit the renormalization-group (RG) approach. In its standard application the RG approach sums up the singularities of PT and provides the power-law behavior of physical quantities which is characteristic of critical phenomena. Here we deal instead with a stable superfluid phase, for which exact cancellations (instead of resummations) of singular terms are expected to occur in physical response functions. It appears thus crucial to exploit the underlying (local-gauge) symmetry and the related Ward identity (WI) which implement these exact cancellations, as required on physical grounds. In this paper we combine the RG approach with the WI to obtain the solution to the problem [14].

To be more explicit we use the WI to (i) establish constraints on the RG procedure, (ii) relate renormalization parameters to physical quantities, and (iii) achieve the cancellation of singularities in the response functions [15]. In this way, the number of marginal and relevant running couplings (which are *a priori* necessary to study the IR behavior of interacting bosons for $d \leq 3$) is reduced to only one, e.g., the longitudinal two-point vertex function Γ_{ll}. In addition, we are able to close the equation for Γ_{ll}, thus providing the *exact* IR behavior for the zero-temperature interacting Bose system. The resulting solution is quite different from the Bogoliubov one despite the coincidence of the linear spectrum. In particular, we free the Gavoret and Nozières results from the provisions posed by the occurrence of IR divergences [3] and recover the result of Ref. [5] for the anomalous self-energy.

We consider the following action for the Bose system:

$$S = \int_0^\beta d\tau \int d\mathbf{x} \Big\{ \psi^*(x)[-\partial_\tau + \mu]\psi(x) - |(\nabla - i\mathbf{A})\psi(x)|^2 - \frac{v}{2}|\psi(x)|^4 + \psi(x)\lambda^*(x) + \psi^*(x)\lambda(x) \Big\}, \tag{1}$$

where $\psi(x)$ [with $x = (\tau, \mathbf{x})$] is a bosonic field obeying periodic boundary conditions in the imaginary time τ and β is the inverse temperature (we set $\hbar = 1$ and $m = 1/2$). In (1) $\lambda(x)$ and $(\mu(x), \mathbf{A}(x)) = A_\nu(x)$ ($\nu = 0, \ldots, d$) are external "sources" introduced to obtain the correlation functions. At the end of the calculation $\mu(x)$ recovers the constant value μ of the chemical potential, while λ and $\mathbf{A}$ are left to vanish. The interaction potential v is taken to be constant in momentum space, as its momentum dependence is found to be irrelevant for the IR behavior.

The action (1) allows for spontaneous broken gauge symmetry; in that case, it is convenient to distinguish between longitudinal (ψ_l) and transverse (ψ_t) components to the broken-symmetry direction by setting $\psi(x) = \psi_l(x) + i\psi_t(x)$ and $\psi^*(x) = \psi_l(x) - i\psi_t(x)$ (with a real order parameter).

By differentiating the free energy $F[A_\nu, \lambda_i] = \ln \int \mathcal{D}\psi_l \mathcal{D}\psi_t \exp\{S\}$ with $i = l, t$ and $\lambda = \lambda_l + i\lambda_t$, we obtain, as usual, the connected correlation functions, such as the "condensate wave function" $\psi_{i0} = \langle\psi_i(x)\rangle = \delta F/\delta\lambda_i(x)$ and the one-particle Green function $\mathcal{G}_{ij} = \delta^2 F/\delta\lambda_i\delta\lambda_j$. It is further convenient to introduce the Legendre transform of F with respect to λ, $\Gamma[A_\nu, \psi_{i0}] = \int dx\,\lambda_i(x)\psi_{i0}(x) - F[A_\nu, \lambda_i]$, whose derivatives are the vertex functions $\Gamma_{i_1\cdots i_n;\nu_1\cdots\nu_m} = \delta^{(n+m)}\Gamma/\delta\psi_{i_1 0}\cdots\delta\psi_{i_n 0}\delta A_{\nu_1}\cdots\delta A_{\nu_m}$ associated with the one-particle irreducible diagrams of PT.

In the broken-symmetry phase we keep the value of the condensate $\langle\psi_l(x)\rangle_{\lambda=0} = \psi_{l0}$ fixed. Accordingly, we introduce the fields $\tilde\psi_i$ with vanishing averages for vanishing external sources, such that $\psi_l(x) = \psi_{l0} + \tilde\psi_l(x)$ and $\psi_t(x) = \tilde\psi_t(x)$. The mean-field propagators $\mathcal{G}_{ij}$ are then obtained from the quadratic part of the action $S_Q = -(1/2)\sum_k \tilde\psi_i(-k)Q_{ij}(k)\tilde\psi_j(k)$, where

$$Q(k) = 2\begin{pmatrix} 3v\psi_{l0}^2 - \mu + \mathbf{k}^2 & -\omega_n \\ \omega_n & v\psi_{l0}^2 - \mu + \mathbf{k}^2 \end{pmatrix}, \quad (2)$$

with $k = (i\omega_n, \mathbf{k})$ (ω_n being a Matsubara frequency). In the following we consider the zero-temperature limit where ω_n becomes a continuous variable ω. Enforcing the mean-field Bogoliubov condition $\psi_{l0}^2 = \mu/v$ yields the IR behavior $\mathcal{G}_{tt} \sim (\omega^2 + c_0^2\mathbf{k}^2)^{-1}$, $\mathcal{G}_{lt} \sim \omega(\omega^2 + c_0^2\mathbf{k}^2)^{-1}$, and $\mathcal{G}_{ll} \sim \mathbf{k}^2(\omega^2 + c_0^2\mathbf{k}^2)^{-1}$, where $c_0 = \sqrt{2\mu}$ is the mean-field value of the sound velocity. This singular IR behavior is associated with the presence of the Goldstone mode. Recall that in the standard ψ representation the Bogoliubov propagators $[\mathcal{G}_{11}(k) = \mathcal{G}_{22}(-k) = \mathcal{G}_{tt}(k) + \mathcal{G}_{ll}(k) - 2i\mathcal{G}_{lt}(k)$ and $\mathcal{G}_{12}(k) = \mathcal{G}_{21}(k) = -\mathcal{G}_{tt}(k) + \mathcal{G}_{ll}(k)]$ share a *common* $(\omega^2 + c_0^2\mathbf{k}^2)^{-1}$ IR behavior and the normal $[\Sigma_{11}(k) = 2\mu]$ and the anomalous $[\Sigma_{12}(k) = \mu]$ self-energies satisfy the Hugenholtz-Pines (HP) identity $\Sigma_{11}(0) - \Sigma_{12}(0) = \mu$ [16]. In the $\psi_{l,t}$ representation, on the other hand, the various propagators have *different* IR behavior since the Goldstone-mode singularity is kept in the transverse direction. This choice is crucial to select the interaction terms according to their relevance [12].

To allow for the RG treatment, it is convenient to rewrite the matrix $Q(k)$ in a more general form:

$$Q(k) = \begin{pmatrix} v_{ll} + z_{ll}\mathbf{k}^2 + u_{ll}\omega^2 & v_{lt} + w_{lt}\omega \\ v_{lt} - w_{lt}\omega & v_{tt} + z_{tt}\mathbf{k}^2 + u_{tt}\omega^2 \end{pmatrix}, \quad (3)$$

where additional terms (running couplings) have been introduced with respect to (2). We also introduce running couplings for cubic $(v_{ttt}, v_{ltt}, \ldots)$ and quartic $(v_{tttt}, v_{lttt}, v_{lltt}, \ldots)$ interaction terms, where the cubic terms originate from the presence of the condensate. A perturbative expansion is then set up, as usual, by regarding the quadratic action associated with (3) as the free part and the remaining terms as perturbations. In the absence of external sources v_{tt} and v_{lt} vanish by symmetry, as shown below. The resulting PT, being massless, is plagued by IR divergences already at the one-loop level in spatial dimension $d \leq 3$ [2,3].

Renormalization of the IR divergencies requires a preliminary power counting for the running couplings. This is conveniently done by keeping dimensionless the minimal set of couplings (v_{ll}, w_{lt}, z_{tt}) that yields the linear part of the Bogoliubov spectrum. We thus rescale them and the fields by appropriate powers of c_0. In this way, $[\mathcal{G}_{tt}] = -2$, $[\mathcal{G}_{lt}] = -1$, and $[\mathcal{G}_{ll}] = 0$, where $[A]$ stands for the engineering dimensions of A. For simplicity, from now on we shall omit indicating c_0 whenever not strictly necessary. We thus have $[\mathbf{x}] = -1$, $[\tau] = -1$, $[\psi_l(x)] = (d+1)/2$, $[\psi_t(x)] = (d-1)/2$, and $[\Gamma^{(n_l+n_t)}_{l\cdots lt\cdots t}(k_1,\ldots)] = -n_l(d+1)/2 - n_t(d-1)/2 + d + 1$. The upper critical dimension is $d_c = 3$ [17]. For $d \leq 3$ the running couplings controlling the IR behavior have dimensions $[v_{ll}] = [w_{lt}] = [u_{tt}] = [z_{tt}] = 0$, $[v_{ltt}] = \epsilon/2$, and $[v_{tttt}] = \epsilon$, with $\epsilon = 3 - d$. Although v_{lt}, v_{tt}, and v_{ttt} would be strongly relevant, they vanish identically for vanishing external sources.

One could proceed at this point and derive the RG equations for the running couplings. As mentioned above, however, contrary to critical phenomena in the present case of a stable phase, a singular PT has to result in finite response functions. It is clear that cancellations of IR divergences in physical quantities signal definite connections among the running couplings. In the present context these connections stem from the local gauge symmetry and are obtained by examining the associated Ward identities [15].

In our formalism the WI result from the local-gauge invariance of the functional Γ, namely,

$$\Gamma[A_\nu + \partial_\nu\alpha(x), R_{ij}[\alpha(x)]\psi_{j0}] = \Gamma[A_\nu, \psi_{i0}], \quad (4)$$

$R_{ij}(\alpha)$ being the two-dimensional rotation matrix with angle α in the space of the fields ψ_l and ψ_t. This equation follows from the invariance of the action (1) under the gauge transformation $\psi(x) \to e^{i\alpha(x)}\psi(x)$, $\lambda(x) \to e^{i\alpha(x)}\lambda(x)$, and $A_\nu(x) \to A_\nu(x) + \partial_\nu\alpha(x)$ with $\alpha(x)$ real function. Taking successive functional derivatives of (4) with respect to α, ψ_{0i}, and A_ν yields an infinite set of WI. For our purposes only the following five WI are relevant. The first two, which encompass the HP identity, are

$$\Gamma_{tl}(k)\psi_{l0} + \Gamma_t(0) - ik_\nu\Gamma_{l;\nu}(-k) = 0\,, \quad (5)$$

$$\Gamma_{tt}(k)\psi_{l0} - \Gamma_l(0) - ik_\nu\Gamma_{t;\nu}(-k) = 0\,. \quad (6)$$

In the limit $k_\nu \to 0$ they relate the two-point vertices to the external sources $\lambda_i = \Gamma_i(0)$ and state that $v_{lt} = \Gamma_{lt}(0)$ and $v_{tt} = \Gamma_{tt}(0)$ vanish when $\lambda_i = 0$. No gap thus appears in the one-particle spectrum. The second couple of WI,

$$\Gamma_{ttl}(k_1,k_2)\psi_{l0} + \Gamma_{tt}(-k_2) - \Gamma_{ll}(k_1 + k_2) - i(k_1)_\nu\Gamma_{tl;\nu}(k_2, -k_1 - k_2) = 0\,, \quad (7)$$

$$\Gamma_{ttt}(k_1,k_2)\psi_{l0} - \Gamma_{lt}(-k_2) - \Gamma_{lt}(k_1 + k_2) - i(k_1)_\nu\Gamma_{tt;\nu}(k_2, -k_1 - k_2) = 0\,, \quad (8)$$

are the standard WI associated with the continuity equation, modified now by the presence of the three-point vertices. In the limit $k_1 = 0$ and $k_2 \to 0$ they yield $v_{ltt}\psi_{lo} = v_{ll}$ and $v_{ttt} = 0$. The last of our WI is

$$\Gamma_{tttt}(k_1, k_2, k_3)\psi_{l0} - \Gamma_{ltt}(-k_2 - k_3, k_2) - \Gamma_{ltt}(k_1 + k_3, k_2) - \Gamma_{ltt}(k_1 + k_2, k_3) - i(k_i)_\nu \Gamma_{ttt;\nu}(k_2, k_3, -k_1 - k_2 - k_3) = 0\,, \quad (9)$$

from which we obtain $v_{tttt}\psi_{l0}/3 = v_{ltt}$ for vanishing k's.

From the above WI we also relate the running couplings to the (composite) current vertices and response functions. Specifically, in the limit $k \to 0$ we obtain

$$w_{lt} = \frac{1}{\psi_{l0}} \frac{\partial^2 \Gamma}{\partial \psi_{l0} \partial \mu}\,, \quad u_{tt} = -\frac{1}{\psi_{l0}^2} \frac{\partial^2 \Gamma}{\partial \mu^2}\bigg|_{\psi_{l0}}, z_{tt} = \frac{2n_s}{\psi_{l0}^2}\,, \quad (10)$$

where n_s is the superfluid density (Josephson identity). We also have $v_{ll} = (\partial^2 \Gamma / \partial \psi_{l0}^2)_\mu$ by its very definition [18]. We are left eventually with four running couplings, namely, v_{ll}, w_{lt}, u_{tt}, and z_{tt}, whose IR behavior can be obtained exactly.

As a guide to the procedure for obtaining this behavior, the RG equations will be evaluated at the one-loop level. Exploiting the ϵ expansion and the minimal subtraction technique we get

$$s\frac{dv_{ll}}{ds} = \frac{c\, v_{ll}^2}{2\bar{\psi}_{l0}^2\, z_{tt}^2}\,, \quad s\frac{dw_{lt}}{ds} = \frac{c\, v_{ll}\, w_{lt}}{2\bar{\psi}_{lo}^2\, z_{tt}^2}\,,$$
$$s\frac{du_{tt}}{ds} = -\frac{c\, w_{lt}^2}{2\bar{\psi}_{l0}^2\, z_{tt}^2}\,, \quad s\frac{dz_{tt}}{ds} = 0\,, \quad (11)$$

where $s = \kappa'/\kappa$ is the scaling factor (κ being the normalization point), $c(s)^2 = v_{ll}(s)z_{tt}(s)/[v_{ll}(s)u_{tt}(s) + w_{lt}(s)^2]$ is the square velocity entering the one-particle propagator according to (3) and $\bar{\psi}_{l0}(s) = \psi_{l0}\kappa^{\epsilon/2}s^{\epsilon/2}$ [19]. When $d = 3$ Eqs. (11) are equivalent to those of Ref. [12], provided the coupling of Ref. [12] analogous to v_{tttt} is identified with $3v_{ll}/\psi_{l0}^2$, consistently with the above results.

Quite generally, the solutions of the coupled equations (11) take the form

$$z_{tt}(s) = z_{tt}(1) \quad w_{lt}(s) = \frac{w_{lt}(1)}{v_{ll}(1)} v_{ll}(s)\,,$$
$$u_{tt}(s) = u_{tt}(1) + \frac{w_{lt}(1)^2}{v_{ll}(1)} - \frac{w_{lt}(1)^2}{v_{ll}(1)^2} v_{ll}(s)\,. \quad (12)$$

This implies that it is sufficient to determine $v_{ll}(s)$.

Although we have derived (12) at the one-loop order, we *expect* them to hold exactly on physical ground owing to the identification (10) of the renormalization parameters with physical quantities. To begin with, the $s \to 0$ value of z_{tt} is the ratio n_s/n_0 of *finite* physical quantities ($n_0 = \psi_{l0}^2$ being the condensate density) so that divergences compensate each other in its expression, leading to the first of (12). Regarding the second of (12), we obtain from (10) for w_{lt} and the definition of v_{ll} that the ratio v_{ll}/w_{lt} reduces to $-2n_0/(dn_0/d\mu)_\lambda$ in the limit $k \to 0$. Here $(dn_0/d\mu)_\lambda$ is the "condensate compressibility" which has to be finite for thermodynamic stability. Finally, from the definition of $c(s)^2$ and (10) we obtain that $c(s)^2$ reduces to c^2 in the limit $s \to 0$, where now $c^2 = 2n_s/(dn/d\mu)_\lambda$ is the square of the macroscopic sound velocity (n being the density and $n = n_s$ at zero temperature). By the very stability of the bosonic system, c^2 is free from IR divergences, thus suggesting that $c(s)$ is finite and does not scale with s, i.e., $c(s) = c = c_0$, apart from finite corrections originating from nonsingular terms that do not enter the RG flow. Exploiting the first and second of (12), we verify that the condition $c(s) = \text{const}$ reduces to the third of (12).

The *proof* of the above statements is as follows. z_{tt} has to remain constant by inspection of the WI (6), which shows that the divergence of z_{tt} expected by power counting is actually not present, since it is related to nondiverging quantities. w_{lt} can instead be identified with v_{ll} via the WI (7) and (8), which relate v_{ll} to Γ_{ltt} and w_{lt} to $\Gamma_{tt;0}$, respectively, the latter identification being obtained from the ω derivative of (8). By inspection of the leading singular terms to all orders in PT, $\Gamma_{tt;0}$ and Γ_{ltt} are then found to be proportional to each other [20]; by the same procedure, the invariance of $c(s)$ implied by the last of (12) follows from the exact connection between the singular parts of $\Gamma_{;00}$ and Γ_{ll}, associated, respectively, with u_{tt} and v_{ll}.

Determining the IR behavior is thus exactly reduced to solving for a *single* running coupling, for instance v_{ll}. In particular, at the one-loop order we obtain

$$\frac{v_{ll}(1)}{v_{ll}(s)} = \begin{cases} 1 - \frac{v_{ll}(1)}{2\bar{\psi}_{l0}(1)^2 z_{tt}(1)^2} \ln s & (\epsilon = 0)\,, \\ 1 + \frac{v_{ll}(1)(s^{-\epsilon} - 1)}{2\bar{\psi}_{l0}(1)^2 z_{tt}(1)^2 \epsilon} & (0 < \epsilon < 2)\,. \end{cases} \quad (13)$$

In both cases $v_{ll} \to 0$ as $s \to 0$, while for $\epsilon < 0$ v_{ll} remains finite. We show below that the asymptotic behavior (13) of v_{ll} is actually exact.

The one-particle Green function resulting from (12) and (13) have the form [19]

$$G_{ll}(k) = \left\{ \begin{array}{ll} -\frac{c\, n_0 \ln k}{64\pi^2 n_s^2} & (\epsilon = 0) \\ \frac{c\, n_0 K_{4-\epsilon}}{8\epsilon n_s^2} k^{-\epsilon} & (0 < \epsilon < 2) \end{array} \right\} \sim \frac{1}{v_{ll}}\,,$$

$$G_{lt}(k) = \frac{dn_0}{d\mu} \frac{c^2}{4n_s} \frac{\omega}{\omega^2 + c^2\mathbf{k}^2} \sim -\frac{w_{lt}}{v_{ll}} \frac{1}{z_{tt}} \frac{\omega}{\mathbf{k}^2 + \omega^2/c^2}\,,$$

$$G_{tt}(k) = \frac{c^2 n_0}{2n_s} \frac{1}{\omega^2 + c^2\mathbf{k}^2} \sim \frac{1}{z_{tt}} \frac{1}{\mathbf{k}^2 + \omega^2/c^2}\,,$$

where the asymptotic ($k \to 0$) values of the running couplings have been identified via (10). Note that the IR behavior of G_{lt} and G_{tt} is completely and exactly determined by the conditions (12) and is independent from d and the behavior of v_{ll}. Instead G_{ll} diverges logarithmically as $k \to 0$ when $d = 3$ and like $k^{-\epsilon}$ for $1 < d < 3$

[5,6,8]. Accordingly, we find that the anomalous self-energy $\Sigma_{12}(k)$ vanishes like $1/\ln k$ or like k^{ϵ}, with the HP identity now reading $\Sigma_{11}(0) = \mu$ [5]. In addition to recovering the result by Gavoret and Nozières [3] for the propagators in the ψ representation, we have also obtained the divergent subleading *ln* terms [21]. Such *ln* terms, on the other hand, can be shown to disappear altogether in the expressions for the density-density, density-current, and current-current response functions [5,20].

Although $v_{ll}(s)$ has been explicitly obtained at the one-loop order, we show now that its asymptotic behavior is exact. In fact, for $d > 3$ all interactions are irrelevant, the Bogoliubov result is correct and no longitudinal divergence appears. For $d = 3$, all perturbation couplings to the Bogoliubov action are marginally irrelevant [12] and the result obtained at the one-loop order is stable. For $1 < d < 3$, the one-loop calculation presented above leads to a nontrivial fixed point [i.e., $v_{tttt}(s)/\kappa^{\epsilon} \to v^* \neq 0$] with $v_{ll} \sim s^{\epsilon}$. The WI result $v_{ll} = v_{tttt}\bar{\psi}_{l0}(1)^2\kappa^{-\epsilon}s^{\epsilon}/3$ guarantees that the exponent of the asymptotic behavior of v_{ll} remains ϵ, *irrespective* of the actual value v^* of the nontrivial fixed point, provided it exists. That k^{ϵ} is actually the true asymptotic behavior of $\Gamma_{ll}(k)$ follows by expressing the singular part of Γ_{ll} in terms of the exact Γ_{ttl} and $\mathcal{G}_{tt}$:

$$\Gamma_{ll}(k) = B - \frac{v_{ll}^0}{2}\sum_q \mathcal{G}_{tt}(q)\mathcal{G}_{tt}(q + k)\Gamma_{ttl}(q, -q - k), \tag{14}$$

where B coincides with the bare coupling $v_{ll}^0 = v_{ll}(s = 1)$ apart from finite contributions. The WI (7) allows one eventually to express Γ_{ttl} in terms of Γ_{ll} itself, thus closing Eq. (14) and providing the exact asymptotic behavior $\Gamma_{ll}(k) \sim k^{\epsilon}$. An analogous procedure was implemented in Ref. [5].

Some final comments are in order. Contrary to the theory of critical phenomena, where the WI do not provide stringent conditions owing to the genuine divergences of physical quantities (such as the compressibility in the gas-liquid transition), in our case the relevant physical quantities are finite and the WI imply cancellations in the corresponding couplings. In this way, lines of fixed points have to appear corresponding to different finite values of the thermodynamic derivatives. It is also worth noticing that the Bogoliubov mean-field solution for $1 < d \leq 3$ is infinitely distant (in the RG sense) from our nontrivial fixed point whenever the interparticle interaction is nonvanishing. This is true despite the fact that the resulting physical picture contains the *same* low-lying elementary excitations.

One of us (F. P.) gratefully acknowledges partial financial support from Europa Metalli-LMI S. p. A. and from the Italian INFM through the Unità di Camerino.

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