Heat-transport Ward identity and effective Landau Fermi-liquid parameters in disordered systems

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The renormalization of the specific heat and of the thermal-diffusion coefficient due to the electron-electron interaction in the presence of disorder is determined by exploiting the skeleton structure of the heat-response function in conjunction with the constraints originating from energy conservation (in the form of a Ward identity). This analysis completes for the thermal properties the identification of a renormalized skeleton theory (suitable for disordered interacting-electron systems) with an effective Landau Fermi-liquid theory.

The theoretical framework of the metal-insulator transition in the presence of disorder has been drastically changing in the last few years with respect to the original independent-particle localization theory (Anderson transition),[1] owing to the inclusion of interaction effects.[2] In fact, while for noninteracting particles the particle-diffusion coefficient only needs to be renormalized, inclusion of interaction effects requires other physical quantities (or couplings) to be renormalized. In particular, the spin susceptibility and the electronic specific heat (that characterize the parameters of the Landau theory of Fermi liquids) acquire singular corrections that are logarithmically divergent in two dimensions.[3]

In a series of papers,[4–6] the additional renormalization parameters of the interacting theory (namely, the singlet Γ_s and the triplet Γ_t interaction amplitudes and a frequency or temperature renormalization parameter z originating from the corrections introduced in the diffusive mode by the electron-electron interaction) have been identified in terms of physical quantities.

(i) In the spirit of the Landau theory of Fermi liquids, the number and spin-density dynamical response functions and the associated conservation laws (in the form of Ward identities) have been analyzed in conjunction with the skeleton structure of the perturbation theory, to show that the linear combinations

$$z_1 = z - \nu\zeta^2(\Gamma_s - \Gamma_0) = \left(\frac{\partial n}{\partial \mu}\right)_T \nu^{-1} , \tag{1}$$

$$z_2 = z - \nu\zeta^2\Gamma_t = \chi_S^{\rm st}/\chi_S^0 \tag{2}$$

renormalize the frequency of the number and spin-density diffusive modes, respectively.[4,5] In Eqs. (1) and (2) ν is the Landau density of states in the absence of disorder (with the proper replacement of the free-electron mass by the effective mass), Γ_0 is the statically screened Coulomb interaction amplitude, $(\partial n/\partial\mu)_T$ is the compressibility per unit density, and $\chi_s^{\rm st}$ is the static spin susceptibility ($\chi_s^0 = \nu\mu_B^2$ is its Pauli counterpart).[7] The parameters z and ζ (together with the diffusion coefficient D) characterize the dressed diffusion propagator [see Eq. (11) below] whereby there is no energy exchange between the electron and the hole: z dresses the frequency of the diffusion mode and ζ the amplitude of the corresponding propagator. Moreover, the particle- and spin-diffusion coefficients are given by

$$D_n = D/z_1, \quad D_s = D/z_2 . \tag{3}$$

(ii) Direct perturbative evaluations of the electronic specific heat c_V (Ref. 6) (at constant volume) and more generally of the heat-response function[8] at first order in the disorder strength but to all orders in the interaction, in direct analogy with Eqs. (1), (2), and (3) have shown that

$$z = c_V/c_V^{(L)} , \tag{4}$$

$$D_Q = D/z , \tag{5}$$

$c_V^{(L)}$ being the (Landau) electronic specific heat in the absence of disorder. A combination of Eqs. (4) and (5) with the Einstein relation then gives for the thermal conductivity $\kappa = c_V D_Q = c_V^{(L)} D$, thereby yielding a generalization of the Wiedemann-Franz law in the presence of interaction:[8]

$$\kappa/T\sigma = c_V^{(L)}/T\nu , \tag{6}$$

where T is the temperature and σ is the electrical conductivity.

In this paper, the renormalization of the specific heat due to the electron-electron interaction in the presence of disorder is identified quite generally in terms of the renormalization parameter z of the frequency of the diffusive mode, by exploiting the skeleton structure of the perturbation theory for the *heat-response function* and the associated Ward identity. In this way, the identification of the three parameters z, z_1, and z_2 is placed on the same footing, thereby settling definitively the correspondence with the Landau Fermi-liquid picture. At the same time, we confirm that the heat-response function possesses a diffusive behavior with diffusion coefficient given by the skeleton perturbation theory according to Eq. (5).

To prove Eqs. (4) and (5) we have to overcome the technical difficulties of dealing with heat transport. We characterize the relevant thermal properties of the system in terms of the heat-response function ($\hbar=1$),

$$\chi_Q(\mathbf{r}-\mathbf{r}';t-t')=-i\theta(t-t')\langle[\mathcal{H}(\mathbf{r},t),\mathcal{H}(\mathbf{r}',t')]\rangle \ , \qquad (7)$$

where $\mathcal{H}(\mathbf{r},t)$ is the grand-canonical Hamiltonian density[9]

$$\mathcal{H}(\mathbf{r},t)=\tfrac{1}{2}\sum_{\xi}\Psi_\xi^\dagger(\mathbf{r},t)[i\partial_t+h_0(\mathbf{r})]\Psi_\xi(\mathbf{r},t) \qquad (8)$$

and $h_0(\mathbf{r})=-\nabla^2/2m+u(\mathbf{r})-\mu$ is the single-particle term in the presence of a random impurity potential $u(\mathbf{r})$ specified by the customary delta-correlated distribution (ξ being a spin label). In Eq. (7) the average over the impurity distribution is understood to be taken after the thermal average, and the time dependence is introduced via the Heisenberg picture of the field operators.

The static limit of the Fourier transform of Eq. (7) provides the specific heat at low temperatures

$$c_V=-(1/T)\lim_{\mathbf{q}\to 0}\lim_{\omega\to 0}\chi_Q(\mathbf{q};\omega)\equiv-(1/T)\chi_Q^{\mathrm{st}} \ , \qquad (9)$$

while the reverse limit determines the thermal conductivity

$$\kappa=-(1/T)\lim_{\omega\to 0}\lim_{\mathbf{q}\to 0}(\omega/\mathbf{q}^2)\mathrm{Im}[\chi_Q(\mathbf{q};\omega)] \ . \qquad (10)$$

For the interacting disordered system we are interested in, the heat-response function is naturally partitioned into a *static* [Eq. (9)] and a *dynamic* part according to the symbolic representation of Fig. 1. Here $\overline{\Gamma}=\Gamma_s-\Gamma_0$ is the proper singlet interaction amplitude that excludes insertion of a Coulomb line, Λ_Q is the heat vertex that renormalizes the bare frequency vertex of the noninteracting case, and L stands for the dressed diffusion propagator between electron and hole lines with positive (+) and negative (−) frequencies

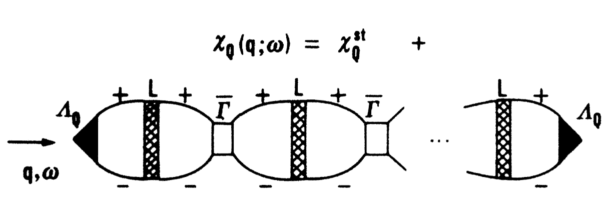

FIG. 1. Skeleton structure for the dynamic part of the heat-response function.

$$L(\mathbf{q};\omega)=\frac{\zeta^2}{-iz\omega+D\mathbf{q}^2+1/\tau_{\mathrm{ph}}} \ . \qquad (11)$$

It has been shown[4,5] that the wave-function renormalization ζ of Eq. (11) can be identified with N/ν, where N is the single-particle density of states in the presence of disorder. It has also been shown[8,10] that the dephasing time τ_{ph}, which would apparently lead to violation of number, spin, and energy conservation, eventually disappears from these response functions. The dynamic part of the heat-response function can then be cast in the following form:

$$\chi_Q^{\mathrm{dyn}}(\mathbf{q};\omega)=-c_V^{(L)}T[\Lambda_Q i\omega\zeta^2\Lambda_Q/(-i\omega z+D\mathbf{q}^2)] \ , \qquad (12)$$

whereby as usual the skeleton structure of Fig. 1 has been handled as an algebraic equation. Notice that z instead of z_1 enters the denominator of Eq. (12) since the presence of a frequency at each vertex makes the amplitude-ladder resummation negligible.

Global number and energy conservation, in the form $\chi_Q(\mathbf{q}=0;\omega)=0$, can be combined with Eqs. (9) and (12), to yield the diffusive behavior

$$\chi_Q(\mathbf{q};\omega)=-Tc_V[D_Q\mathbf{q}^2/(-i\omega+D_Q\mathbf{q}^2)] \qquad (13)$$

with D_Q given by Eq. (5) and with the condition

$$\zeta^2\Lambda_Q^2=zc_V/c_V^{(L)} \ . \qquad (14)$$

An additional condition is required to identify both Λ_Q and z in terms of physical quantities. Akin to the cases of number and spin diffusion,[5] a Ward identity associated now with local energy conservation provides this additional condition. To this end, we extend to the interacting case the Ward identity introduced for the noninteracting system.[11] We consider the temperature function corresponding to Eq. (7) and its Fourier transform

$$\chi_Q(\mathbf{q};\Omega_\lambda)=-\int_0^{(k_BT)^{-1}}d(\tau_1-\tau_2)e^{i\Omega_\lambda(\tau_1-\tau_2)} \times\int d\mathbf{r}_1 e^{-i\mathbf{q}\cdot(\mathbf{r}_1-\mathbf{r}_2)} \times\sum_{\xi_1\xi_2}\langle T_\tau[\mathcal{H}'(1)\mathcal{H}'(2)]\rangle \ , \qquad (15)$$

where T_τ is the imaginary-time ordering, $\mathcal{H}'(1)=\mathcal{H}(1)-\langle\mathcal{H}(1)\rangle$, 1, . . . stand for the set of space, spin, and imaginary-time variables, and $\Omega_\lambda=2\lambda\pi k_BT$ (λ integer) is a Matsubara frequency. We also introduce the vertex

$$\Lambda(1;2,2')=\langle T_\tau[\mathcal{H}'(1)\Psi^\dagger(2')\Psi(2)]\rangle \qquad (16)$$

which, at given configuration of disorder, is related to χ_Q by

$$\chi_Q(1,2)=-\tfrac{1}{2}[-\partial_{\tau_2}+h_0(\mathbf{r}_2)] \times[\Lambda(1;2,2,')-\tfrac{1}{2}\delta(1,2')G(2,1)]_{2'=2^+} -\delta(1,2^+)[\langle\mathcal{H}(1)\rangle+\tfrac{1}{2}\langle\mathcal{H}_{\mathrm{int}}(1)\rangle] \ , \qquad (17)$$

where G is a temperature single-particle Green's function and $\langle\mathcal{H}_{\mathrm{int}}(1)\rangle$ is the average interaction energy per unit volume. The vertex $\Lambda(1;2,2,')$ satisfies the following equation:

$$\partial_{\tau_1}\Lambda(1;2,2')=\langle T_\tau[\partial_{\tau_1}\mathcal{H}'(1)\Psi^\dagger(2')\Psi(2)]\rangle +\delta(1,2')\partial_{\tau_1}G(2,1) +\delta(1,2)\partial_{\tau_1}G(1,2')+\cdots , \quad (18)$$

where the additional terms denoted by the ellipsis at the right-hand side of Eq. (18) depend explicitly on the interaction potential and drop out upon integration over the variables $\mathbf{r}_1$ and ξ_1 (which corresponds to setting $\mathbf{q}=0$ in the Fourier transform). Making use of the continuity equation

$$\partial_{\tau_1}\mathcal{H}'(1)+\nabla_1\cdot\mathbf{J}'_Q(1)=0 , \quad (19)$$

where $\mathbf{J}'_Q(1)$ is a suitably defined heat current operator, Eq. (18) transforms into the following Ward identity:[12]

$$\int d\mathbf{r}_1\sum_{\xi_1}\partial_{\tau_1}\Lambda(1;2,2') =\int d\mathbf{r}_1\sum_{\xi_1}[\delta(1,2')\partial_{\tau_1}G(2,1)+\delta(1,2)\partial_{\tau_1}G(1,2')] . \quad (20)$$

We notice at this point that the dynamic part of χ_Q originates only from the first term at the right-hand side of Eq. (17). We then replace in this term $\frac{1}{2}[-\partial_{\tau_2}+h_0(\mathbf{r}_2)]$ by $-\partial_{\tau_2}$, perform the impurity average, and take the Fourier transform. The key point is now to realize that, by restricting the frequency summation in this Fourier transform according to the + and − structure of Fig. 1, we effectively suppress one of the two vertices Λ_Q of the related expression (12) for χ_Q^{dyn}, as derived from perturbative analysis.[8] We can thus make use of the Ward identity (20), to obtain

$$\chi_Q^{\mathrm{dyn}}(\mathbf{q}=0;\Omega_\lambda) =\Lambda_Q\frac{2k_BT}{\Omega_\lambda}\sum_{m=-\lambda}^{-1}\omega_m^2 i\int\frac{d\mathbf{p}}{(2\pi)^d}[G_-(\mathbf{p};\omega_m) -G_+(\mathbf{p};\omega_m+\Omega_\lambda)] =\Lambda_Q c_V^{(L)}TN/\nu \quad (\Omega_\lambda\to 0) . \quad (21)$$

A combination of Eq. (21) with Eq. (12) then enables us to relate Λ_Q and z to the wave-function renormalization $\zeta=N/\nu$, giving the required condition

$$\zeta\Lambda_Q=z . \quad (22)$$

From Eqs. (14) and (22) we identify eventually z with the specific-heat renormalization according to Eq. (4).

We have in this way completed the identification of the renormalization parameters of the theory of disordered interacting electronic systems, in terms of scale-dependent Landau Fermi-liquid parameters. We have achieved this goal by exploiting the skeleton structure of the number, spin, and heat-response functions together with the constraints supplied by the corresponding global and local conservations. In all cases, global conservation leads to the diffusive behavior of the response functions while the associated Ward identities connect the dynamic part of the response functions with the single-particle density of states.

Finally, the validity of the Wiedemann-Franz law (6) (in the presence of strong disorder and including the electron-electron interaction) finds here a simple physical explanation since we have been assuming a collisionless regime[13] of the effective Landau quasiparticle picture.

[1]See, e.g., *Anderson Localization,* edited by Y. Nagaoka and H. Fukuyama (Springer, New York, 1982).

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[7]z_1 does not get renormalized by disorder and coincides with its Landau value $z_1^0=1-\nu(\Gamma_s^0-\Gamma_0^0)=(1+F_0^s)^{-1}$. z_2 becomes instead scale dependent and is strongly modified with respect to its Landau expression $z_2^0=1-\nu\Gamma_t^0=(1+F_0^a)^{-1}$. See Refs. 4 and 5.

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[9]The form (8) for the grand-canonical Hamiltonian density is not Hermitian since Ψ and $\Psi^\dagger$ appear in an asymmetric way. This is not going to affect our results in the limit of vanishing wave vector in the Fourier transform of Eq. (8) we shall consider.

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[12]A Ward identity for heat diffusion has been considered by J. S. Langer [Phys. Rev. **128**, 110 (1962)] who made use of it in the reverse limit $\Omega_\lambda=0$, $\mathbf{q}\to\mathbf{0}$.

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